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XXIX. *Note on the Connexion between the Method of Least Squares and the Fourier Method of Calculating the Coefficients of a Trigonometrical Series to represent a given Series of Observations of a Periodic Quantity.* By CHARLES H. LEES, D.Sc., F.R.S.

RECEIVED MAY 6, 1914.

1. In view of the fact that several methods of representing a number of observations of a variable quantity by means of a trigonometrical series of the Fourier type have been proposed which differ from that adopted by Fourier himself, it seems advisable to call attention to a characteristic of the Fourier method of determining the coefficients of the terms of such a series, which gives them a greater degree of reliability than the coefficients determined by any other method. This property they owe to the close connexion between the method of their calculation and the method of least squares as used in the solution of a number of simultaneous equations which exceeds the number of independent variables to be determined.

Although this connexion is referred to by Kelvin and Tait * it is seldom mentioned in recent books† on the subject of Fourier series, which concentrate attention mainly on the validity of the expansion when the number of terms is infinite and there are no errors of observation.

The connexion ought not to be lost sight of, as it serves not only as a good introduction to the Fourier method of expansion, but also exhibits very clearly how the expressions used in calculating the coefficients of the terms of the series arise.

2. Let it be required to determine the coefficients in the expansion of a periodic function $f(x)$ of x of period l in a finite series of the form

$$A_1 \sin 2\pi x/l + \dots + A_g \sin 2g\pi x/l + \dots + A_n \sin 2n\pi x/l + B_0/2 + B_1 \cos 2\pi x/l + \dots + B_g \cos 2g\pi x/l + \dots + B_n \cos 2n\pi x/l \quad (1)$$

and let the values of $f(x)$ be known for q values of x —i.e., $x_1, x_2, \dots, x_p, \dots, x_q$ —in the interval $0 < x < l$, where $p < q$ and q is considerably greater than $2n+1$.

* "Treatise on Natural Philosophy," Vol. 1, art. 398, p. 456.

† See Hobson, "Theory of Functions"; Bromwich, "Theory of Infinite Series"; Thomae, "Bestimmte Integrale und Fouriersche Reihen"; Carslaw, "Fourier Series and Integrals"; Byerly, "Fourier Series."

Then, if the errors in the values of $f(x)$ follow the normal law of errors, we get the most probable values of the coefficients by the method of least squares*—that is, we take those values of the coefficients which make the sum of the squares of the q differences between $f(x)$ and the series at the points $x_1, x_2 \dots x_p \dots x_q$ a minimum.

In symbols,

$$\sum_{p=1}^q \{f(x_p) - \sum_{g=0}^n (A_g \sin 2g\pi x_p/l + B_g \cos 2g\pi x_p/l)\}^2$$

is to be made a minimum by a proper choice of the coefficients A and B .

Differentiating with respect to each of the coefficients A_k, B_k , where $k \leq n$, we have for a minimum or a maximum

$$\sum_{p=1}^q \{f(x_p) - \sum_{g=0}^n (A_g \sin 2g\pi x_p/l + B_g \cos 2g\pi x_p/l)\} \sin 2k\pi x_p/l = 0$$

and

$$\sum_{p=1}^q \{f(x_p) - \sum_{g=0}^n (A_g \sin 2g\pi x_p/l + B_g \cos 2g\pi x_p/l)\} \cos 2k\pi x_p/l = 0. \quad (2)$$

Differentiating again we see that the preceding equations give the conditions to be fulfilled for a minimum, and the $2n+1$ equations of type (2) determine the $2n+1$ coefficients required.

3. If the q points $x_1, x_2 \dots x_p \dots x_q$ are uniformly distributed over the range 0 to l , we may write $x_p = lp/q = hp$, say, where $l/q = h$, and the equations determining the coefficients become

$$\sum_{p=1}^q \{f(ph) - \sum_{g=0}^n (A_g \sin 2g\pi ph/l + B_g \cos 2g\pi ph/l)\} \sin 2k\pi ph/l = 0$$

and

$$\sum_{p=1}^q \{f(ph) - \sum_{g=0}^n (B_g \sin 2g\pi ph/l + B_g \cos 2g\pi ph/l)\} \cos 2k\pi ph/l = 0, \quad (3)$$

or

$$\sum_{p=1}^q f(ph) \sin 2k\pi ph/l = \sum_{p=1}^q \sin 2k\pi ph/l \sum_{g=0}^n (A_g \sin 2g\pi ph/l + B_g \cos 2g\pi ph/l)$$

and

$$\sum_{p=1}^q f(ph) \cos 2k\pi ph/l = \sum_{p=1}^q \cos 2k\pi ph/l \sum_{g=0}^n (A_g \sin 2g\pi ph/l + B_g \cos 2g\pi ph/l). \quad (4)$$

* For laws of error for which this would not be the case see Edgeworth's article on "Probability" in the "Encyclopædia Britannica," Vol. 22, p. 397.

On interchanging the order of summation the two right members become

$$\sum_{g=0}^n \{A_g \sum_{p=1}^q \sin 2k\pi ph/l \cdot \sin 2g\pi ph/l + B_g \sum_{p=1}^q \sin 2k\pi ph/l \cdot \cos 2g\pi ph/l\},$$

and

$$\sum_{g=0}^n \{A_g \sum_{p=1}^q \cos 2k\pi ph/l \cdot \sin 2g\pi ph/l + B_g \sum_{p=1}^q \cos 2k\pi ph/l \cdot \cos 2g\pi ph/l\}. \quad (5)$$

On expressing each product as a sum or difference of the sines or cosines of the sum and difference of the angles, it is seen that the sums with regard to p are zero except in the case $g=k$, when terms $\frac{1}{2} \cos 0^\circ$ lead to the sum $q/2$.* Hence we have

$$\sum_{p=1}^q f(ph) \cdot \sin 2k\pi ph/l = A_k q/2$$

and

$$\sum_{p=1}^q f(ph) \cdot \cos 2k\pi ph/l = B_k q/2.$$

That is,

$$A_k = (2/q) \sum_{p=1}^q f(ph) \cdot \sin 2k\pi ph/l$$

and

$$B_k = (2/q) \sum_{p=1}^q f(ph) \cdot \cos 2k\pi ph/l. \quad \dots \quad (6)$$

4. If the number q of observations is very large h becomes very small, and on writing x for ph and dx for h the equations become

$$A_k = (2dx/l) \sum_{p=1}^q f(x) \cdot \sin 2k\pi x/l,$$

$$\text{and } B_k = (2dx/l) \sum_{p=1}^q f(x) \cdot \cos 2k\pi x/l$$

or, in the notation of the calculus,

$$A_k = (2/l) \int_0^l f(x) \cdot \sin 2k\pi x/l \cdot dx,$$

$$\text{and } B_k = (2/l) \int_0^l f(x) \cdot \cos 2k\pi x/l \cdot dx. \quad (7)$$

It is thus seen that the method of least squares leads to the usual Fourier expressions (6) and (7) for the coefficients of the series (1), and shows very clearly how the sine and cosine terms come into them. By the theory of probability if the errors of observation follow the normal law the coefficients determined by (6) or (7) are the most probable.

* See Fourier, "Théorie analytique de la Chaleur," Art. 269.

ABSTRACT.

In view of the number of alternative methods which have been suggested for calculating the coefficients of the terms of a trigonometrical series to represent a number of observations of a periodic quantity, the author points out that the Fourier method gives the most probable values of the coefficients, since it makes the sum of the squares of the errors at the points of observation a minimum.

DISCUSSION.

Dr. C. CHREE mentioned that the method was dealt with in Tait's "Natural Philosophy," but no proof was given. Prof. Lees had supplied the proof, and the Paper was of interest on that account.

Dr. W. WILSON (communicated remarks) said: Prof. Lees deals with a function $f(x)$ of period l , whose values are given in the whole interval from 0 to l . In this case the validity and *uniqueness* of the Fourier expansion (provided $f(x)$ is subject to certain restrictions) have been demonstrated with complete rigour (Dirichlet, "Collected Works," Vol. I., pp. 133-160; and G. Cantor, "Journal für Mathematik," LXXII.). It seems to me, therefore, that there is no question of the reliability of the Fourier coefficients, and the fact that the method of least squares leads to the usual Fourier expressions for the coefficients confirms the reliability of this method rather than that of the Fourier coefficients themselves.

Prof. LEES (communicated): I think the reference mentioned by Dr. Chree is the one cited in my Paper, but since the Paper was read Sir Joseph Larmor has told me he believes Sir John Herschel dealt with the subject in one of his Papers, which I have not yet succeeded in finding. I am sorry anything in my Paper should have led Dr. Wilson to think I was dealing with the infinite series of Dirichlet and others whose work is embodied in the text-books cited. n is restricted to finite values, and in nearly all actual cases is a very small integer. Thus the independent simultaneous equations whose number q exceeds that of the $2n+1$ variables do not admit of a *unique*, but only of a *most probable*, solution. Hence the alternatives to the Fourier solution suggested at various times by *e.g.*, Wedmore, Lyle, Fischer-Hinnen and Thompson.

XXX. *A Magnetograph for Measuring Variations in the Horizontal Intensity of the Earth's Magnetic Field.* By F. E. SMITH, A.R.C.Sc. (*From the National Physical Laboratory.*)

IN March of this year Dr. C. Chree read a Paper before this Society on "Time Measurements of Magnetic Disturbances"* in which he discussed the question of the simultaneity of occurrence of magnetic disturbances over the whole earth. The records examined by Dr. Chree resulted from the use of at least five different patterns of magnetographs: the Mascart and Eschenhagen, with very small magnets; the Watson, with composite systems formed of a number of parallel magnets; the Kew, with magnets weighing about 87 grammes; and the Greenwich, which stand to the Kew somewhat as these stand to the Eschenhagen. If one wished for comparative times on quick-run curves correct to within a few seconds, Dr. Chree said it would be advisable, if not absolutely necessary, to employ instruments of the same pattern, similarly sensitive. This highly desirable condition of affairs may not result in the immediate future, but if the movement is helped forward by this description of the magnetograph in use at the National Physical Laboratory our main object will be attained.

In all instruments known to the author for recording the variations in the horizontal intensity of the earth's magnetic field, a magnet, or system of magnets, is suspended by a fibre, or fibres, the torsion on which is sufficient to deflect the magnetic system to a position nearly at right angles to the meridian.

In the case of unifilar instruments, if θ is the angle which the magnetic system makes with the meridian, M the moment of the magnet, and H the horizontal intensity of the earth's field, the couple $MH \sin \theta$ is balanced by the torsional couple of the fibre. If T is the torsional couple for 1° of twist and ϕ is the angle through which the upper end of the fibre is turned with reference to the lower end, we have

$$MH \sin \theta = T\phi. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Normally, declination and horizontal intensity are changing simultaneously so that θ changes wholly irrespective of

* "Proc." Phys. Soc., Vol. XXVI., p. 137, 1914.

changes in H or φ . For the purpose of comparing sensitivities of instruments with various values of θ and φ we may, however, suppose the declination to remain constant. The manner of calculating the changes in H when the declination varies is considered later.

When the declination is constant and H changes to $H - \partial H$ with a corresponding deflection $\partial\varphi$ of the magnetic system, we have

$$M(H - \partial H) \sin(\theta + \partial\varphi) = T(\varphi - \partial\varphi) \quad \dots \quad (2)$$

and

$$\frac{\partial H}{H} = \cot \theta d\varphi + \frac{\partial\varphi}{\varphi} \quad \dots \quad (3)$$

For purposes of calculation it is convenient to regard the angles as measured in degrees. We then have

$$\frac{\partial H}{H} = 0.0175 \partial\varphi \cot \theta + \frac{\partial\varphi}{\varphi} \quad \dots \quad (4)$$

and

$$\frac{\partial\varphi}{\partial H} = \frac{\varphi}{H[1 + 0.0175 \varphi \cot \theta]} \quad \dots \quad (5)$$

$\cot \theta$ is necessarily small, and the term including it is not of great importance unless φ is very great. In most instruments used in observatories, φ is about 60° , and in such cases sufficient precision is often obtained by neglecting the $\cot \theta$ term and writing (5) in the form

$$\partial\varphi = \varphi \partial H / H \quad \dots \quad (6)$$

The same relation is obtained by letting $\theta = 90^\circ$ and differentiating equation (1). The sensitiveness of such instruments is directly proportional to the torsion on the fibre.

In the instruments we have experimented with φ has been made very great. In the general case it was of the order of $5,000^\circ$, but in one case it exceeded $100,000^\circ$. Since in practice θ is never 90° (except by accident, for the system is unstable when θ has this value) the term involving $\cot \theta$ becomes of considerable importance.

The following table shows the variation of the sensitiveness with variation of φ and θ . The angle of deflection $\partial\varphi$ of the magnetic system is given when $\partial H/H = 1/18,000$, i.e., for a change in the horizontal intensity from $H - 0.5\gamma$ to $H + 0.5\gamma$, the total change in H being 1γ .* For convenience we give also

* $1\gamma = 0.00001$ of the C.G.S. unit of intensity of magnetic field.

the displacement d in millimetres at a distance of 2 metres of a reflected ray of light.

θ .	$\phi=60^\circ$.		$\phi=100^\circ$.		$\phi=1,000^\circ$.		$\phi=5,000^\circ$.		$\phi=10,000^\circ$.	
	$\partial\phi$.	d in mm.	$\partial\phi$.	d in mm.	$\partial\phi$.	d in mm.	$\partial\phi$.	d in mm.	$\partial\phi$.	d in mm.
89°	$0.20'$	0.2_3	$0.32'$	0.3_8	$2.6'$	3.0	$6.6'$	7.7	$8.2'$	9.6
88°	$0.19'$	0.2_3	$0.31'$	0.3_7	$2.1'$	2.4	$4.1'$	4.8	$4.7'$	5.5
87°	$0.19'$	0.2_2	$0.31'$	0.3_6	$1.7'$	2.0	$3.0'$	3.5	$3.3'$	3.8
86°	$0.19'$	0.2_2	$0.30'$	0.3_5	$1.5'$	1.7	$2.4'$	2.7	$2.5'$	2.9
85°	$0.18'$	0.2_1	$0.29'$	0.3_4	$1.3'$	1.5	$1.9'$	2.3	$2.1'$	2.4
84°	$0.18'$	0.2_1	$0.28'$	0.3_3	1.2	1.4	$1.7'$	1.9	$1.7'$	2.0
83°	$0.18'$	0.2_1	$0.28'$	0.3_2	$1.1'$	1.2	$1.4'$	1.7	$1.5'$	1.8
82°	$0.17'$	0.2_0	$0.27'$	0.3_1	$1.0'$	1.1	$1.3'$	1.5	$1.3'$	1.5

The particular object of the table is to show how the sensitiveness varies with change of θ and ϕ . *The values tabulated must not be used to convert measured ordinates into changes of force.* For such conversion the ordinates are measured from a fixed base line representing a known angular position from the magnetic meridian, and the corresponding changes in H are deduced from Fig. 1. This method will now be considered (1) when H varies but the declination is constant, and (2) when H and D vary simultaneously.

Subject to the condition that the declination is constant, the variation of θ and of ϕ with change in H is shown graphically in Fig. 1. If we suppose that initially θ is exactly 90° , we have

$$MH = T\phi.$$

If H changes to $H+h$ and α is the corresponding deflection, we have as an exact relation

$$\frac{h}{H} = \left(\frac{1}{\cos \alpha} - 1 \right) + \frac{\alpha}{\phi \cos \alpha}.$$

As ordinates we have plotted $\left(\frac{1}{\cos \alpha} - 1 \right)$ and $\alpha/\phi \cos \alpha$ for values of α between 0° and 8° , corresponding to a change in θ from 90° to 82° . The ordinates to the curve OA from the base OP represent the values of $\left(\frac{1}{\cos \alpha} - 1 \right)$; they are to be considered as positive. The ordinates to the curves OB , OC , OD , OE and OF from the same base OP represent the corresponding values of $\alpha/\phi \cos \alpha$ for initial values of the torsion of 60° , 100° , $1,000^\circ$, $5,000^\circ$ and $10,000^\circ$ respectively. The chart enables one to find the change in H necessary to deflect the magnetic

needle from any one position to any second position conditionally that both positions are between 82° and 90° from the magnetic meridian. Thus for an instrument with $1,000^\circ$ of torsion on the fibre an increase in H of 20 parts in 10,000, *i.e.*, 36γ , is required to deflect the magnet from a position 87° from the meridian to a second position 86° from the meridian. The increase in H is given by the difference between the ordinates KL and GH . The figure shows that even for

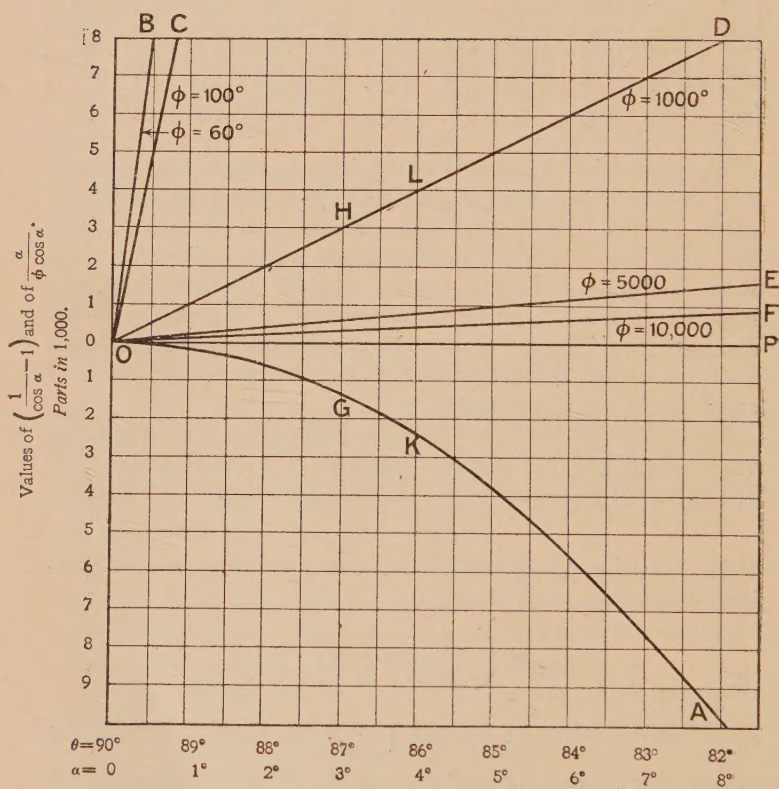


FIG. 1.

measurements of very small variations no great advantage is gained by increasing the torsion beyond $5,000^\circ$. With the latter value for ϕ a decrease in H of four parts in 10,000 (*i.e.*, about 7γ) suffices to deflect the magnet from 89° to 90° , and so disarrange the system. While, therefore, such an instrument is admirable for measuring very small and sudden variations

in the horizontal intensity, it cannot be used for measuring very large fluctuations unless the sensitiveness is reduced by reducing θ .

As previously stated, declination (D) and horizontal intensity (H) usually change simultaneously, so that θ changes independently of variations in H or ϕ . To determine the real changes in H, it is necessary to measure the changes in D, and it is desirable to have the D and H variations recorded on one sheet, the scale values (in degrees) of the D and H records being identical or nearly so (*i.e.*, the D and H instruments should be equally distant from the recording drum).

Initially we may suppose the base line record of the H instrument to coincide with the H record when θ is 90° . In practice there are several ways of doing this. One of the simplest is to, at first, convert the H instrument into a declination instrument by freeing the fibre from torsion. The magnetic system is then deflected through an angle θ (say 89°) by producing torsion on the fibre equal to $\phi - \beta$ where β is the extra torsion required to make $\theta = 90^\circ$ and $\phi = (\phi - \beta) / \sin \theta$. The mirror used for the base line record is now adjusted to produce a record in the position corresponding to $\theta = 90^\circ$, and the position of the record of the second instrument (the declination instrument) is noted at the same time. When thus set up, the records obtained enable one to read off d (the change in declination from the initial declination and which may be plus or minus), and α the angular displacement of the magnetic system from $\theta = 90^\circ$.

Let θ and D be regarded as measured from geographical north. Then, when the magnetic system is at right angles to the magnetic meridian $(\theta - D) = 90^\circ$ and

$$MH \sin (\theta - D) = MH = T\phi.$$

When H [the value of the horizontal intensity when $(\theta - D) = 90^\circ$] changes to $H + h$ and the declination increases by an amount d , we have

$$M(H + h) \sin [(\theta - D) - (\alpha + d)] = T(\phi + \alpha)$$

where α is the angular displacement of the magnetic system and is given by the distance of the H record from the base line.

The above equation reduces to

$$\frac{h}{H} = \left[\frac{1}{\cos (\alpha + d)} - 1 \right] + \frac{\alpha}{\alpha + d} \cdot \frac{\alpha + d}{\phi \cos (\alpha + d)},$$

and we are enabled readily to determine the value of $\frac{h}{H}$ from Fig. 1. Thus suppose $\alpha=3^\circ$, $d=+1^\circ$, and $\varphi=5,000^\circ$. Then

$$\frac{h}{H} = \left[\frac{1}{\cos 4^\circ} - 1 \right] + \frac{3}{4} \left[\frac{4^\circ}{5,000^\circ \cos 4^\circ} \right].$$

The values of the quantities in brackets are given in Fig. 1. We have

$$\frac{h}{H} = [0.00244] + \frac{3}{4} [0.00080],$$

i.e., $\frac{h}{H} = 0.00304$, or $h = \text{about } 55\gamma$ if $H = 0.18$.

As a summary, we may say "Whatever may be the angular deflection (α) of the magnetic system from the initial position corresponding to $\theta - D = 90^\circ$, we add (or subtract) the change in declination, d , from it. To the ordinate of the OA curve (Fig. 1) corresponding to $\alpha + d$ we add $\frac{d}{\alpha + d}$ times the ordinate of the φ curve. This gives the change in the horizontal intensity.

I believe that most recording instruments in constant use have a sensitiveness such that a change of 5γ in H produces a deflection of the magnetic system of about 1 minute of arc. The corresponding displacement of a spot of light at a distance of 2 metres is about 1 millimetre. It is therefore difficult, and in some cases impossible, to determine even from high-speed records, the magnitude of very small and sudden changes.

The design of a magnetograph for detecting such changes engaged my attention some time ago. My main object was to detect with certainty sudden changes in the horizontal intensity and to measure these changes within 0.5γ .

The form of instrument which I have used is shown in Fig. 2. It differs from other instruments in general use inasmuch as the swinging system is critically aperiodic and the damping is effected by air. It may be used for detecting changes as small as 0.1γ , or it may be employed as a less sensitive instrument for producing records of changes extending over many years. The weight of the suspended system is usually less than 1 gram. The instrument is cheap to construct, is portable, and is easy to set up.

The details of construction are as follows. A fine quartz

fibre F is suspended from a torsion head H, and supports two aluminium vanes v and V , a concave mirror m and a magnet M . The magnetic system is attached to the back of the mirror and may consist of one or several small magnets. The lightest magnetic system I have used consisted of one magnet 2 mm. long and about 0.5 mm. in diameter. The weight of the magnet was a little less than 0.01 gram; together with the mirror and aluminium vane the weight was 0.1 gram. The heaviest system consisted of 6 magnets, each of which was 2.5 cm.

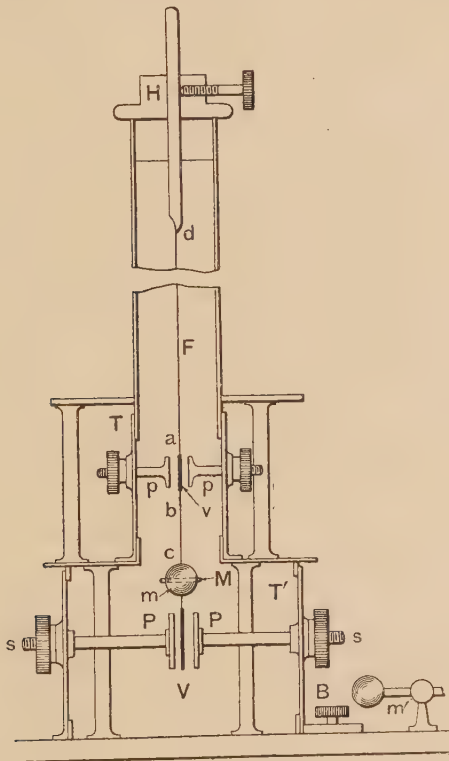


FIG. 2

long and about half a millimetre in diameter. This system was at times loaded with lead to increase the moment of inertia, and the total weight was then about 50 grams. In general the magnetic system was chosen to make the value of φ between $3,000^\circ$ and $5,000^\circ$.

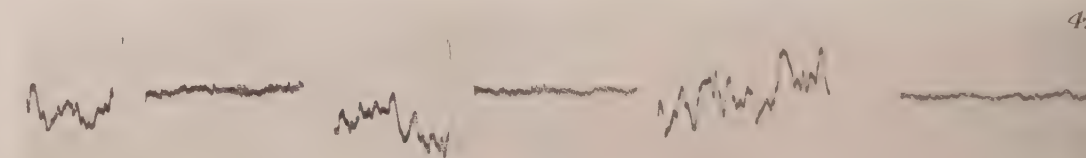
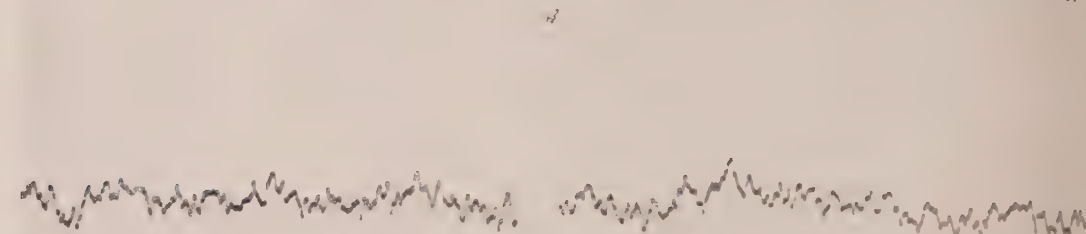
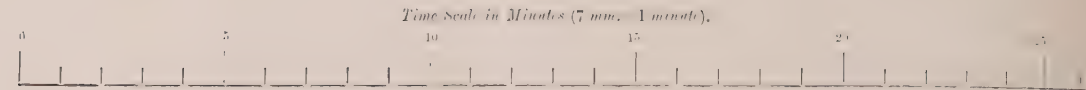
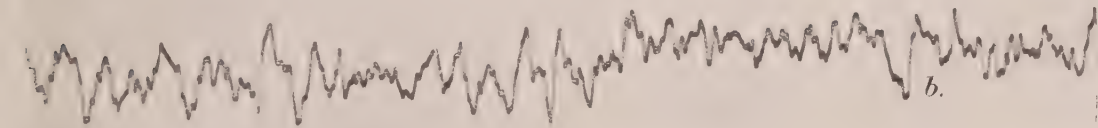
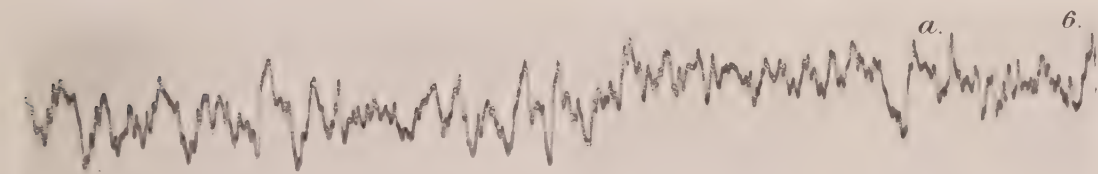
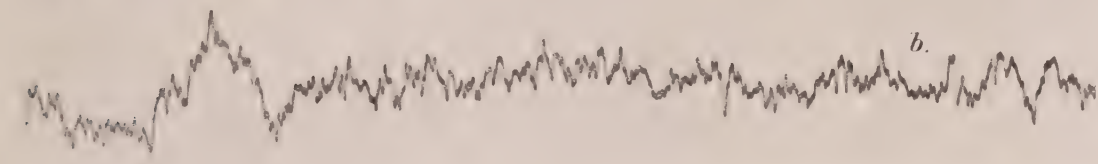
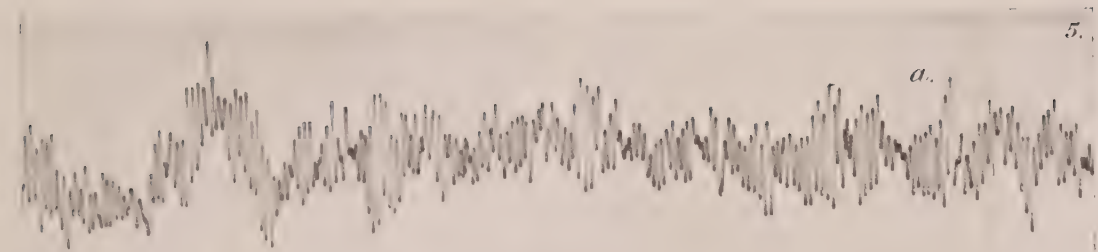
PP are damping plates of aluminium, their distance apart

being controlled by the screws *ss*. *pp* are a similar but smaller pair of plates.

The damping vanes *v* and *V* are of thin aluminium foil, *v* being of circular shape with two projecting ears, thus \bigcap_b^a ; the diameter of the vane is greater than the diameter of one of the plates *pp*. The quartz fibre is laid across the vane in the direction *ab*, and secured by shellac to the projecting ears. Afterwards the short length of fibre between *a* and *b* is removed.

The tube *T* supporting the plates *pp* is free to rotate and is provided with two small circular windows to enable the vane and plates to be viewed. When the vane is clamped by screwing the plates into firm contact with it the length of fibre controlling the magnet is that between the points *b* and *c*. When the vane is free to move the effective length of the fibre is increased to *bc* plus *ad*. The relative sensitivities of the two arrangements depends on the ratio of *bc* to *bc+ad*. In practice, suppose the more sensitive arrangement to be in use and that it is desired to make the instrument less sensitive. The position of the spot on the recording drum having been observed, the tube *T* is rotated until the plates *pp* are approximately parallel to *v*; *v* is then clamped. Should the position of the spot have been disturbed the tube *T* may be rotated until the position is the same as before. ϕ has now been reduced to $\phi bc/(bc+ad)$ (this supposes the fibre to be uniform in thickness) and θ remains constant. The change occupies but one or two minutes. Of course if an instrument of high sensitiveness only is desired the plates *pp* and the vane *v* are omitted.

In our instruments the vane *V* was in general about 1 cm. long and 1 cm. broad, but in special experiments the length of the vane was increased to 5 cm. When the axis of the magnetic system is displaced by torsion so as to lie in a plane nearly 90° from the meridian, the vane *V* swings approximately in the meridian. The tube *T'* which supports the damping plates *PP* may be rotated through about 20° in order to ensure that the surfaces of the plates are approximately parallel to the vane. The tube is then clamped in position by the screw *B*. There are two circular windows which enable the vane and plates to be viewed, and through one of these the light is transmitted to the mirror *m*. A fixed mirror, *m'*, is supported by a stiff ball and socket joint and reflects the light for a base line record.



To face page 287.]

To determine the sensitiveness of the instrument* a current of 0.002 ampere is passed through two coaxial circles of wire (not shown in the figure), the circles being 35 cm. in diameter and 3 cm. apart. The planes of the circles are at right angles to the meridian, and the centre of the magnet M is on the line joining the centres of the circles, or nearly so. The two circles of wire are supported by the base of the instrument. When making a determination of the sensitiveness the current is passed in the same direction through the two circles of wire and the deflection of the spot is recorded. The current is then reversed for a record of the deflection in the opposite direction. The sensitiveness is thus determined with ease and precision. It must, however, be remembered that the scale value varies across the recording sheet.

In our experiments two horizontal intensity instruments have been used. One has been erected in a permanent position and has been but little interfered with; the other is a rough model built for experimental work. Both instruments were constructed in the laboratory workshops.

The general usefulness of the instruments is best illustrated by some of the records which have been obtained.

Record 1.—This record shows the normal behaviour of the permanent instrument when the damping is critically aperiodic. The value of ϕ is nearly $5,000^\circ$ and the value of θ is about 86° . The large displacements are the result of passing and reversing a current of 0.002 ampere through the two circles of wire. The mean sensitiveness across the portion of the sheet traversed by the record is 2.3 mm. for a change in H of 1γ . The time scale (abscissa) is such that 430 mm. represent one hour (7 mm. represent 1 minute). The sudden changes of the order of 4γ , which are depicted, are largely due to the earth currents produced by the London United Electric Tramway system. These changes very rarely exceed 5γ .

Record 2.—This shows the usual magnitude of the disturbances when no trams are running, the sensitiveness being the same as before. The record was taken between 3 a.m. and 4 a.m.

Record 3.—It follows from the Table (p. 281) that if θ be increased to 89° a much greater sensitiveness should be obtained. The result of such a change is shown in Record 3. The mean sensitiveness across the portion of the sheet traversed

* It is not necessary to determine the sensitiveness experimentally if ϕ and θ are known.

is such that a change in H of 1γ is represented by a displacement of 8 mm. Several such records, with different values for θ , were taken to check equation (5). The sudden changes shown in Record 3 in no instance exceed 4γ .

Record 4.—This illustrates the change from a sensitiveness of 2.8 mm. displacement for 1γ to 0.31 mm. for 1γ , and *vice-versa*. The time occupied in changing from one system to the other was from 1 to $1\frac{1}{2}$ minutes.

Records 5 to 8.—These records are in pairs and illustrate the importance of correctly damping the system. Two instruments were used to produce a pair of records, the records being simultaneously taken and the sensitivities of the instruments being identical within 5 per cent. The *b* records resulted from an instrument the damping of which was always critically aperiodic.

Records 5a and 5b.—*5a* shows the result of under damping, the damping plates being withdrawn to their maximum extent. The mean sensitiveness of each instrument was 3.0 mm. for a change of 1γ . It will be observed the maximum displacement in *5a* occurs when there are a series of small impulses; these are shown in *5b*. If we attempt to measure the magnitude of the sudden changes in the horizontal intensity from the displacements recorded in *5a* errors of several hundreds per cent. occur.

Records 6a and 6b.—The two instruments were made equally sensitive and both were critically damped. The system for *6a* consisted of six small magnets, a comparatively coarse fibre, and a damping vane about 4 sq. cm. in area. That for *6b* consisted of one small magnet, a fine fibre and damping vane 2 sq. cm. in area. The value of ϕ was about $4,000^\circ$ for *6a* and $3,500^\circ$ for *6b*, and the periods of the undamped systems were eight and seven seconds respectively. The records, together with others, were taken to detect any instrumental peculiarities and to gauge the precision of our measurements. The results are remarkably good and show the instrumental peculiarities to be practically negligible. When these results are considered in conjunction with the data establishing the relationship between the magnitude and duration of a sudden change and the deflection, we conclude that the measurement of a sudden disturbance of 5γ lasting for at least three seconds may be relied on within 0.1γ . The error involved in measuring a disturbance lasting for more than three seconds is even less.

The time occupied in making the suspended system and

setting up the second instrument (for 6*a* records) was about one hour. Clearly it is not difficult to construct instruments to give practically identical results.

Records 7a, 7b and 8a, 8b.—The two instruments were equally sensitive, but one (giving the *a* records) was overdamped. Our main object was to alter the period and damping of one system in order that it would respond to slow variations in the horizontal intensity, but not to sudden oscillatory ones such as those produced by an electric traction system. The mean value of *H* over a period of five minutes or longer is thus more readily deduced. Such an instrument is of considerable value when absolute measurements are being made (as with the Kew magnetometer) for the record shows the slow natural changes but masks the local disturbances. In an attempt to obtain the records 7*a* and 8*a* I at first made the moment of inertia of the suspended system very great to obtain a long time of swing, and I increased the damping by employing a large vane. However, the results were disappointing as the mass of the system was too great for the air damping to be sufficiently effective. A very light suspended system (0.1 gram) was therefore tried with a fine fibre, and a vane of 2 sq. cm. area, and with this excellent results were obtained. The record 7*a* shows very few of the rapid disturbances indicated in 7*b* notwithstanding that the sensitivities of the two instruments were identical. For the record 8*a* the damping was still further increased, and it will be seen from the smooth curve that small disturbances of short duration have practically no effect on the instrument.

For the permanent instrument (when the damping was critically aperiodic) photographs of the damping curve were taken to determine the relationship between the magnitude and duration of a sudden change, and the deflection produced. In addition, artificial disturbances of known magnitude were produced for from $\frac{1}{2}$ to 20 seconds. If *d* is the deflection produced by a disturbance lasting so long that the effect of damping may be neglected, the deflection resulting when the disturbance lasts from $\frac{1}{2}$ second upwards is as follows :—

0.5 second deflection	=	about	32 per cent. of <i>d</i> .
1.0	"	"	55 " "
1.5	"	"	72 " "
2.0	"	"	85 " "
2.5	"	"	95 " "
3.0	"	"	=greater than 95 " "

The magnitude of a disturbance lasting for more than 3 seconds may, therefore, be measured with considerable accuracy. If necessary, the suspended system may be modified in order that a disturbance lasting for 1 second may be accurately measured.

As with other forms of magnetograph, temperature variations produce deflections of the magnetic system, and it is necessary therefore to know the temperature coefficient of the instrument or to keep the temperature constant. A considerable reduction of the temperature coefficient (possibly its elimination) would result by the action of a bimetallic strip on the torsion head. With increase of temperature, ϕ would be diminished, and with decrease of temperature it would be increased. In the near future we hope to experiment with such a compensating device.

ABSTRACT.

In the case of unifilar instruments for recording variations in H , if θ is the angle which the magnetic system makes with the magnetic meridian, M the moment of the magnet, and H the horizontal intensity of the earth's field, equilibrium results when $MH \sin \theta = T\phi$, where ϕ is the torsion on the fibre and T is a constant. In the instrument described ϕ may be made great or small, but high sensitiveness is secured by making ϕ great. The magnet system is supported by a quartz fibre, and critically aperiodic damping is obtained by means of an aluminium vane and two parallel damping plates. To diminish the sensitiveness the effective length of the fibre may be reduced. The general usefulness of the instrument is illustrated by photographic records, which show the instrumental peculiarities to be very small, and indicate that unless the system is aperiodic increased difficulty must result in the interpretation of the records. An over-damped system responds but slightly to rapid pulsations in H , but follows the slow changes which are common all over the world. The general sensitiveness of the records is about 3 mm. for a change in H of 0.00001 C.G.S. unit, but one record shows a displacement of 8 mm. for such a change.

DISCUSSION.

Dr. CHREE thought the instrument ingenious and likely to be very useful for the purpose for which he understood it was primarily intended, viz., the observation of the disturbances produced at the National Physical Laboratory by existing and prospective electrical tramways and railways. There were various features in the existing disturbances whose investigation seemed likely to be of interest and to the public advantage. There were also certain natural phenomena, for example, "pulsations," or small oscillations of magnetic force, for whose investigation the instrument from its great sensitiveness seemed well adapted, only for such purposes it would have to be set up at some station, such as Eskdalemuir, remote from London or any other large centre of electrical industry. There were, however, two features, viz., the somewhat rapid variation of scale value across the sheet, and the fact that the instrument

responded sensibly to changes of declination as well as horizontal force, which, he thought, stood somewhat in the way of its employment for ordinary *observatory* purposes.

Mr. R. S. WHIPPLE asked whether the author had considered the advisability of flashing a spot of light across the sheet to give the time scale, as was done at Potsdam.

Mr. C. W. S. CRAWLEY considered the tramways were a great nuisance to magnetic observers. He had gone a good deal further in sensitiveness than the author, and even when situated 16 miles from the nearest tramways he had found them very troublesome. The little kick in the middle of the disturbance which Mr. Smith thought might be an instrumental error had often been noticed by him, and was a definite phenomenon in no way due to the instrument.

Prof. T. MATHER asked if the author had tried using the finest quartz fibre which he could handle. It appeared that the instrument took two or three seconds to attain its final position. If a finer fibre were used, a lighter and quicker magnetic system could be employed.

The AUTHOR, in reply, agreed with the remarks of Dr. Chree. He had thought it would be interesting to set up an instrument to detect very small pulsations, such as might occur during solar eclipses, while neglecting the larger ordinary disturbances. He admitted that the variation of sensitiveness across the sheet would be a disadvantage to an ordinary observatory assistant, but it gave him no trouble whatever. In reply to Mr. Whipple, he had already started to employ the flash of light method for the time scale. In reply to Prof. Mather, there was no doubt that the system could be made a good deal lighter if it were desired to measure very sudden disturbances lasting only a few seconds.

XXXI. *The Atomic Weight of Copper by Electrolysis.* By
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I. Introduction.

II. Limiting Cathode Current Density (D).

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2. Deposition of Hydrogen.
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Table I. Conditions of Experiment.

Table II. Weighings.

I. INTRODUCTION.

The following is an account of the determination of the relative weights of copper and silver deposited when the same electric current passes through copper and silver voltameters. For the successful determination of the relative weights by the electrical method several important effects have to be borne in mind :—

1. *The solution of the deposited metal* in the liquid of the cell. In a preliminary investigation it was found that if the solution

was assumed to be proportional to the area of the cathode very concordant results were obtained for the corrected values of the ratio of the deposited weights of copper to that of silver. In this way the mean error in the value of the ratio for nine early experiments was altered from 1 part in 500 to 1 part in 6,000. In order to apply the solution correction four copper cells in series, having different cathode areas, were used in the final experiments. By a graphical construction the amount which would have been weighed on a cathode, free from loss by solution, was always taken as the corrected weight of the copper deposited in the cell. In the case of silver this correction was found to be unnecessary, and therefore in the final work the silver was determined by taking the mean of the weights of the deposits of two silver cells, these cells being separated by four others in which copper was deposited.

2. The cathode current density must be kept *below a certain limiting value*, depending upon the concentration of the electrolyte, the amount of acid present and the geometry of the cell. If this value is exceeded the deposit is non-coherent, powdery in consistency and apparently amorphous. Experiment shows that such deposits do not yield concordant results on weighing. In order to determine the limiting current-density a series of experiments in which the conditions were varied systematically was carried out. The chief result obtained was that in all cases the deposition of unweighable metal was heralded by a rise in the reading of a voltmeter placed across the cell. Under the heading of "Limiting Current Density" (Section II.) the method of predetermining the safe current for any cell is explained. In the final determinations the information thus gained is utilised so as to permit the employment of widely varying current densities, all, however, well under the limiting values.

3. A deposit may, however, be unsatisfactory in other ways. Visible crystalline growths, liable to be lost in washing or weighing, may present themselves. Coarse-grained deposits of this type may also do harm by imprisoning some of the electrolyte. The allowable current density in this case depends upon the weight to be deposited—the greater the amount to be deposited the lower must be the current density. By rotating the cathodes, or by using porous pots, both the weight deposited and the current density may be increased. These points are further considered under the heading "Crystalline Growths" (Section III.).

4. Before the final determinations were made the effects of impurities in the cell were investigated. These effects were either nil or were eliminated by using suitable current densities, by rotating the cathodes, or by using porous pots. Details are discussed under the heading "Impurities" (Section IV.).

5. It was found to be essential that the changes in the weights of the deposits, due to exposure in drying and weighing, should be eliminated by equal treatment of all the deposits in each experiment. The graphical method of correcting for loss by solution will then at the same time correct for oxidation, &c., if the washing and drying be done simultaneously for all the deposits.

II. THE LIMITING CATHODE CURRENT DENSITY.

1. *The Cylindrical Cell.*

The deposits obtained in the common types of cells are far from being uniform, especially with parallel plate electrodes, where the deposit on the edges is often granular and non-coherent. The cylindrical cell was finally adopted because it gives uniform deposits and allows, other things being equal, a higher current density to be used. A cell of this type, a copper voltmeter, is shown in the accompanying diagram (Fig. 1). The anode consists of a copper tube. For other metals the anode foil is kept in position by xylonite tubes, which drop into circular slots cut in the base, and are fixed with paraffin wax. The cathode consists of a brass rod. The brass cap, shown in position, need only be used when it is desired to rotate the cathode, the cap and the V-shaped slot then being amalgamated. Sometimes it is necessary to introduce porous partitions between the electrodes. These are the ordinary battery porous pots, the top being cut off to the required length. After being cleansed by boiling successively with aqua regia, potassium cyanide, nitric acid and water, they, too, are dropped into circular grooves and fixed in position with paraffin wax. It is important that there should be no leakage under the pot.

2. *Deposition of Hydrogen.*

Suppose we take the copper cell just described, fix the cathode and add a solution of copper sulphate. Let the current be gradually increased and the deposit examined from time to time. At first the deposit is bright, metallic, coherent and uniform throughout. After a time the deposit immediately below the surface of the electrolyte becomes tarnished

and then quickly develops into a chocolate coloured non-coherent mass. As time proceeds, or the current is raised further still, this non-coherent mass extends from the upper

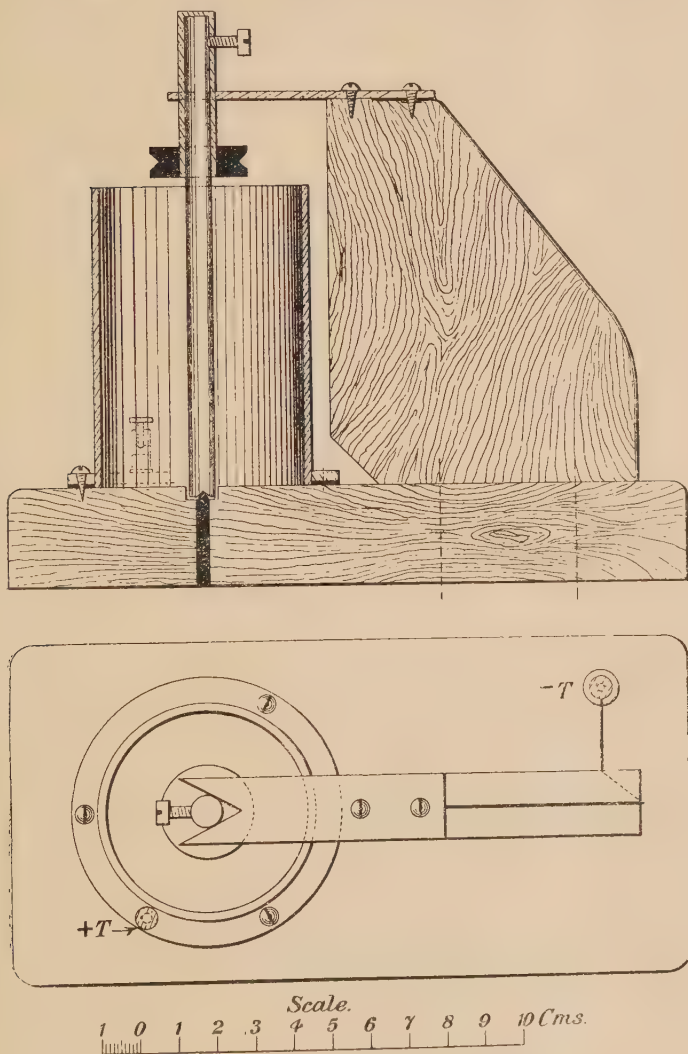


FIG. 1.—THE CYLINDRICAL VOLTAMETER.

part of the electrode downwards, quickly at first and then gradually more slowly. If the current is sufficiently high this

will be succeeded by the evolution of hydrogen. A voltmeter placed across the cell indicates an increase in voltage simultaneously with the deposition of the chocolate mass. This change in the character of the deposit is not due to the anode, since covering it with two or three layers of filter paper makes no difference. Neither is it due to the atmosphere, because in one experiment two identical cells were run in series, one having the electrolyte covered with a layer of oil. In both the effect appeared at the same time and to the same extent. Nor does the amorphous mass consist of copper oxide, since a portion of it, after being cleaned, dried and weighed, was reduced in a stream of hydrogen, and when re-weighed showed no loss in weight. The appearance of this deposit is similar to that of copper precipitated from a solution of copper sulphate by means of zinc. The same phenomenon is observed in other cells, the non-coherent deposit for zinc being bluish grey, for silver a dirty grey and for gold an orange red. In all cases, however, provided that the cathode current density is below a definite value, the action never takes place. On the other hand, if the current density (d) be raised above a certain value, the whole of the current cannot be carried by the copper ions, and some of it must be carried by the hydrogen ions. When the latter are deposited they interfere with the deposition of the copper, and hence give rise to the non-coherent deposit. When the rate of deposition becomes greater hydrogen will be evolved as gas. The copper thus deposited becomes so voluminous that it greatly increases the effective area of the cathode, consequently the cathode current density is reduced below the critical value for the deposition of hydrogen, and the copper is deposited in the metallic state again, the deposit then having the appearance of a gnarled oak stem. Hence, in order that the deposits may yield concordant results, it is necessary that the cathode density should be below a certain maximum or limiting value.

3. *The Methods of Obtaining the Limiting Current Density, "D."*

There are two ways of obtaining the value of D , the maximum or limiting cathode current density, namely, the weighing and voltmeter methods.

(a) *The Weighing Method.*—In a given cell systematically increased currents are run for the same length of time. The weights of the deposits are proportional to the currents. When, however, the copper is deposited by the secondary reaction

the deposit when weighed will no longer be proportional to the current. By a graphical construction this point of departure is found, and gives the maximum current which can be used. From this the limiting current density is deduced. This method is chiefly applicable to the case of rotating cathodes.

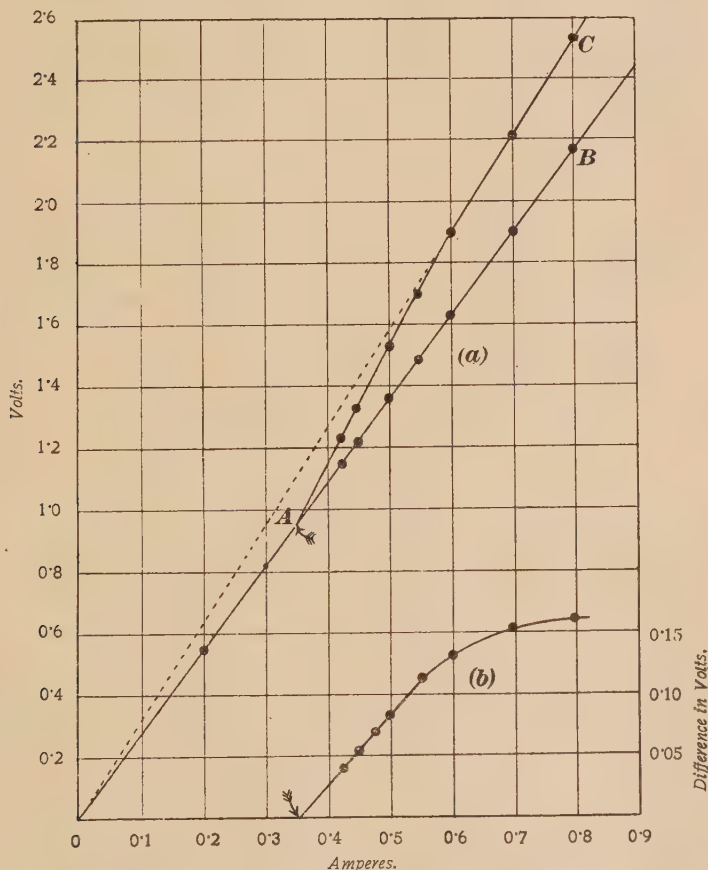


FIG. 2.—THE RISE IN VOLTAGE WHEN THE LIMITING CURRENT IS EXCEEDED.

(b) *The Voltmeter Method*.—A more expeditious method was found by placing a voltmeter across the cell, and noting its readings for steady increments of current. This gives the ordinary ohmic straight line passing through the origin, the line OAB of Fig. 2, which is the graph for one experiment.

But if the current is 0.8 ampere the needle, after standing at B for a time, begins to rise slowly at first, and then with increasing speed till the point C is reached, where it remains stationary or gradually falls. An examination of the cathode shows the presence of the non-coherent deposit. If the current is high enough the needle oscillates about the maximum reading, the amplitude of the oscillation giving a rough measure of the rate of the evolution of hydrogen. The curve passing through the maximum readings cuts the line OAB at A.

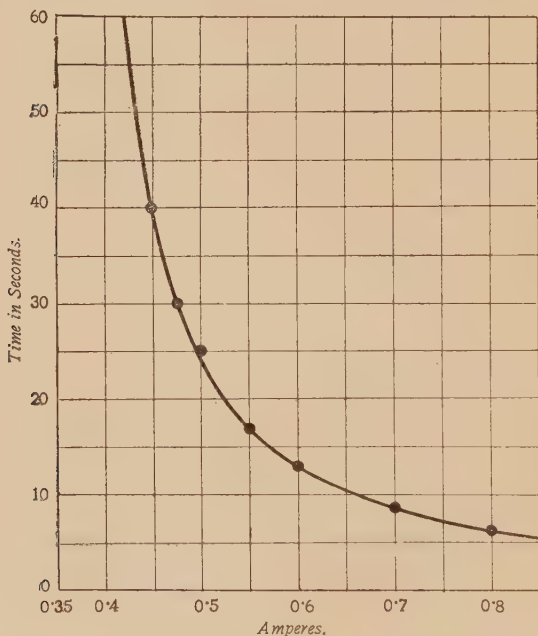


FIG. 3.—TIME FOR THE VOLTAGE TO RISE TO THE HIGHER VALUE WHEN THE LIMITING CURRENT IS EXCEEDED.

Below this point there is no trace of the non-coherent deposit. The point of intersection is taken as the limit to which the cell can be safely run. From this the value of D is deduced. In this experiment the cathode was cleaned before each reading. If this is omitted then the graph consists of the broken line OAC. Fig. 3 is for the same experiment, but in this case the ordinates represent the time in seconds for the voltage to rise to its higher value, after the switching on of the current. This curve can be represented by the equation $T = a(C - C_0)^{-b}$,

where C_0 is the distance of the vertical asymptote from axis of time, *i.e.*, with a current equal to, or below, C_0 the time for secondary deposition to take place is infinite. In this case the value for C_0 , the limiting current, is equal to 0.35 ampere, the same as for the point A in Fig. 2.

The voltmeter method is capable of further modification to meet particular cases. By these methods the value of D was found for the voltameters of copper, silver, gold, and zinc under varying conditions of:—concentration of the electrolyte, height and radial thickness of the cell, change of temperature, the addition of acid and other impurities to the electrolyte, the inclusion of a porous pot, and the rotation of the cathodes.

4. *The Value of the Limiting Current Density, "D."*

The following empirical formula was found to give the limiting cathode current density for the electrolytes copper sulphate, zinc sulphate, silver nitrate, and gold chloride, the last containing acid,

$$D = ab^x c^y d^z, \quad \dots \dots \dots (1)$$

where D is measured in amperes per square centimetre. The values for the quantities are given in the table below.

Cell.	<i>a.</i>	<i>c.</i>	<i>d.</i>	<i>x.</i>	<i>b.</i>	<i>y.</i>	<i>z.</i>
Cu	0.0525	0.94	1.06	1.11	Concentration of the electrolyte in terms of normal strength.	Height of the cathode in cs. between the limits 3-7 cs.	Radial thickness of the electrolyte in cs. between the limits 2-4 cs.
Zn	0.0534	0.94	1.06	1.09			
Ag	0.094	0.94	1.06	1.33			
Au	0.050	0.94	1.06	1.14			

In this table *b*, *c*, *d*, *y* and *z* are independent of the particular metal, whilst *a* and *x* are peculiar to each metal. Further, it was found that in the case of Cu, Zn and Ag the limiting current density can be expressed by the formula

$$D = e(1-\alpha)wc^y d^z, \quad \dots \dots \dots (2)$$

in which ab^x in equation (1) is replaced by $e(1-\alpha)w$. Here $e(=2.13)$ is a constant, the same for the three metals, $(1-\alpha)$ is the un-ionised fraction of the salt in solution, and *w* is the mass of the metal per cubic centimetre. The equation (2), written in full, becomes

$$D = 2.13 (1-\alpha)w(0.94)^b(1.06)^{r_1-r_2}. \quad \dots \dots (2A)$$

In 45 cases the calculated and experimental value of D agreed within the limits of experimental error. From equation (2A) it is evident that the value of D depends upon the geometry

of the cell, and also to a large extent upon the density convection currents.

Temperature.—All the above equations apply more particularly to the case in which the temperature of the laboratory and the electrolyte is about 18°C. If, however, the temperature rises, the new value of the constant (e) in equation (2) is given by

$$e_t = e_{18} + 0.05(t - 18^\circ). \quad \dots \dots (3)$$

Porous Pots.—If the electrodes are separated by porous pots the convection currents due to the removal of the metal ion are greatly restricted, and the allowable D falls in consequence. The new value for D is still given by the equations (1) and (2), but (z) is now reduced to zero.

Acids.—The allowable current-density is also cut down by the presence of acid in the cases of Cu, Zn and Ag, and if D be calculated from equations (1) and (2), which is for neutral solutions, the actual allowable current-density is found to be given by the equation

$$D_a = D(0.93)^n, \quad \dots \dots (4)$$

where n is the concentration of the acid in terms of N/10 strength.

Rotation of the Cathodes.—The value of D rose when the cathode was rotated in the case of Cu and Zn, whether a porous pot was used or not, according to the equation

$$D_R = D(1 + 0.0024R), \quad \dots \dots (5)$$

where R is the number of revolutions per minute. The results for silver in this case were not conclusive.

It thus appears that, provided cylindrical cells are used, the maximum safe current can be calculated for all conditions imposed upon the experiment.

III. CRYSTALLINE GROWTHS.

Having arranged that the value of the cathode current-density (d) shall be sufficiently small to satisfy the previous rules, it is still necessary to restrict the total amount of deposit in order to avoid difficulties from crystalline growths. This is given by the simple empirical equation

$$w\Delta = \text{constant}, \quad \dots \dots (6)$$

where w is the total number of grammes deposited on a cathode

and Δ is the maximum current-density at which these crystalline growths are not produced. The value of the constant for different conditions of the cell is given in the accompanying table.

Metal.	Electrolyte.	Porous pot.	Rotation of cathode.	$w\Delta$.
Copper	Neutral	None	Stationary	0.025
	Neutral	Porous pot	Stationary	0.05
	+ $\frac{1}{10}$ % of H_2SO_4 by volume	Porous pot	Stationary	0.025
	Idem	Porous pot	1,000 r.p.m.	0.035
Silver	Neutral	None	Stationary	0.08
	Neutral	None	600 r.p.m.	0.08
	Neutral	Porous pot	700 r.p.m.	0.2

It is necessary, therefore, to use in any experiment a (d) less than either D or the Δ calculated from the above table.

IV. IMPURITIES IN THE CELL.

1. *Anode Detritus.*

Very soon after the current is started a density convection current of the electrolyte is set up. This current, descending at the anode and ascending at the cathode, carries with it anode detritus. This gives rise to coarse coralline deposits at the bottom of the cathode. Covering the anode with a few layers of filter paper makes no improvement. Rotating the cathode diminishes this effect to an extent depending on the rate of revolution, but a trace of it is sometimes seen in the form of a spiral on the rod, which may in silver develop into fan-like plates. The trouble is eliminated by separating the electrodes with a porous pot. In this case copper oxide, sand and pumice powder may be held in suspension in the electrolyte without materially affecting the deposit. It seems, therefore, that only very small particles, which can pass through filter paper but not through a porous pot, can act as nuclei for the formation of the coralline bottom growths.

2. *Acid.*

Acid added to the cell clarifies the electrolyte and decreases the anode detritus. Besides lowering the value of D it modifies the character of the deposit considerably. The normal deposits of copper and silver consist of octahedral crystals. In the case of copper, if the current-density is above 0.01, the

crystalline deposit is destroyed. In its place we have a creamy non-crystalline surface, and on this hemispherical protuberances appear, increasing in number and size with time, till they become contiguous, and then develop into a coralline structure. Deposits from acidified solutions give concordant weighings, provided that the experiment is kept within the $w\Delta$ limits of equation (6). On the other hand, the crystalline deposit of silver is not destroyed, but in the upper region of the cathode needle-shaped crystals soon appear, which, branching, develop into beautiful lace-like patterns. For this reason, in the final determinations, only neutral solutions of silver nitrate were used.

3. *Other Added Impurities.*

The following impurities were introduced in the preliminary experiments to test the effect on the value deduced for the atomic weight. In the copper cell were introduced ZnSO_4 , FeSO_4 , AgNO_3 and CaCl_2 ; and in the silver cell $\text{Cu}(\text{NO}_3)_2$.

(a) The presence of AgNO_3 in the electrolyte was at once detected by the silver being precipitated from the solution on the electrodes. The CaCl_2 immediately blackens the cathode, and causes the needle of a voltmeter when placed across the cell to give very unsteady readings.

(b) The presence of ZnSO_4 , FeSO_4 or $\text{Cu}(\text{NO}_3)_2$ in the electrolyte lowers the value of D the same as acids do according to equation (4).

(c) Neither Zn nor Fe is deposited in the copper cell. In a particular experiment the electrolyte contained equal parts of Cu and Zn. Using a current-density of 0.013, a deposit having a beautiful matt surface was obtained. This was peeled off, and 0.323 gramme of it dissolved. The copper precipitated as $\text{Cu}(\text{CNS})_2$ yielded 0.322 gramme. Clearly, therefore, no zinc was deposited. Graham obtained a similar result with iron, the proportion of Cu to Fe in the electrolyte being 3 to 2.

Neither is Cu deposited from a silver cell. Repeating the experiment of Lord Rayleigh and Mrs. Sidgwick, but having a higher copper content, the proportion of Ag to $\frac{1}{2}\text{Cu}$ being 5 to 1, a good deposit was obtained. 0.9437 grammes of the deposit was dissolved, and the silver, precipitated as AgCl , gave 0.9436 gramme. These experiments cover practically all the impurities likely to be present in the two electrolytes.

4. *The Purity of the Metals and Salts used in the Final Determinations.*

In the final work no difference in the results was obtained when the electrodes consisted of the ordinary laboratory metal, or the extra pure of Kahlbaum, and the same is true for the salts copper sulphate, and silver nitrate.

V. AN ACCOUNT OF PREVIOUS WORK.

The solution of the Cu cathode by the electrolyte was first pointed out by Gore (1). Gray (2) also showed that this effect varied with the different conditions of the cell. Richards and Collins (4) found that the loss in weight of a cathode was proportional to its area. The loss was little affected by the presence of acid, but increased with the concentration of the electrolyte. This confirmed the work of Foerster and Seidel (19). They pointed out that the Cu dissolves in a strong CuSO_4 solution to form Cu_2SO_4 , which may, especially in neutral solutions, yield too high a result, due to the presence of cuprous ions. Schuster (20) also showed that the solvent power of acid in the electrolyte was small. Experiment, however, showed that the solution effect cannot be corrected for by control experiments, especially as Adams and McNutt (3) proved that it is only the freshly deposited Cu which is so sensitive to the action of acid. The graphical method of correcting for solution under experimental conditions showed that the solvent power of the electrolyte does not increase with its concentration, but does increase with the acid content. The effect sometimes encountered with electrolytes previously boiled over Cu, where the weight of Cu deposited increases with the area of the cathode, may be looked upon as due to oxidation, which can be easily corrected for by the graphical method. If, however, Cu_2SO_4 is formed in the cathode space, its concentration and the amount of deposition by the cuprous ions will still be a function of the cathode area. Hence the graphical method of correction will cover this case, too. This method is preferable to that adopted by Beach (5) and Shephard (6), who added arbitrary amounts of ammonium chloride to their neutral electrolytes to prevent oxidation.

The amorphous deposit discussed in section II. was noted as far back as 1844 by Smee (7). Le Blanc (8) considered that both copper and hydrogen ions are present at the cathode, but the latter are not liberated until the concentration of the

copper ions is reduced to zero. Sand (9), using a vertical cell, came to the same conclusion. But Hittorf (10), as quoted by Sand, believed that the two ions are deposited at all current-densities. The hydrogen, however, reacts with copper sulphate with a definite reaction velocity, and is only evolved when its rate of deposition is greater than the reaction velocity. It seems from the experiments on the limiting current-density that more probably a definite cathode fall in potential is necessary for the deposition of hydrogen.

In order to increase the effective cathode current density Graham (11) circulated his electrolyte. Experiment shows, however, that better results are obtained when the cathode is rotated. The rotation of the cathode is employed by Cowper-Coles (12) for the deposition of copper on an industrial scale. He obtains a smooth deposit, which he maintains is due to the skin friction between the electrode and the electrolyte. Smooth deposits were not obtained in my experiments which may be due to the peripheral speed being insufficient. Also to eliminate the coral growths, due to anode detritus, he surrounds the bottom of the cathode with a baffle plate, placed a short distance away from it.

A résumé of the work on the silver voltameter is given by Guthe (13). This contains an account of the Guthe-Richards cell, which was used in the final determinations of the Atomic Weight in this Paper. The work of Lord Rayleigh and Mrs. Sidgwick (17), and that of Smith, Mather and Lowry (18), will be referred to in the next section.

The earlier determinations of the atomic weight of copper, chiefly performed by the reduction of CuO , were vitiated by systematic errors. This was pointed out by Richards (14). In his classic work he estimates the combining weights of Cu and Br in CuBr_2 ; the Cu, two determinations, by its deposition from 50 cc. of solution by electrolysis, and the bromine, six determinations, by precipitating it as AgBr . By combining these results 12 values for the atomic weight are obtained, whose mean is 63.57—the accepted value in the 1913 list of atomic weights, where $\text{O}=16$, $\text{Ag}=107.88$. Shaw (15), by the electrolytic deposition of Cu and Ag from cells in series, the Cu being deposited on 1 to 16 platinum wires, obtained the atomic weight 63.46. He endeavoured to correct for solution as a function of the current density. In the preliminary experiments of this Paper similar corrections gave abortive results. Using a silver cell of the Rayleigh and Sidgwick

type, and two copper cells of the parallel plate type, Richards and Collins (4) obtained the value 63.534, with a mean error of 0.02 for the last six consecutive determinations. This value is for the corrected weight of Cu found by extrapolating to zero area, the same as explained in this Paper. Unfortunately only two plates were used, and no note is given to say if they were simultaneously washed and dried. Subsequent work by Richards and Heinrod (21) showed that the weight of Ag deposited in the cell of the Rayleigh and Sidgwick type was greater than that obtained when a porous pot surrounds the anode. The correction for this raised the atomic weight of copper to 63.560. Also they found that by heating the Ag deposit to redness the deposit lost in weight. This further correction raised the atomic weight to 63.571. Lastly Pécheux from a similar electrolysis with neutral solutions, obtained the value 63.43. No details are given in the Paper referred to (16).

VI. THE ATOMIC WEIGHT OF COPPER.

In the following determinations two silver cells separated by four copper cells were run in series. This will correct for any short circuiting of the current. The ratio of the corrected weight of the deposits of copper to that of silver was found. From this the atomic weight of copper

$$= \frac{\text{Weight of copper}}{\text{Weight of silver}} \times 2 \times 107.88.$$

The solution of the copper deposit was corrected for by the graphical method. This necessitated simultaneous washing and drying of the electrodes. The maximum cathode current density used must be less than that calculated from the equations (1) to (5). Also for a given weight of metal to be deposited the greatest current density used must be less than that given in equation (6).

1. *The Silver Cells.*

The cylindrical cell with a silver rod for cathode allowed only a small range of current densities to be used. As larger cathodes were not at my disposal the Guthe-Richard's cell was used. This is a modification of that of Lord Rayleigh and Mrs. Sidgwick (17). The cathode consists of a platinum dish, which is separated from the anode by a porous pot, the anode plate being tightly packed in with fine grains of silver, chiefly obtained from previous deposits. The silver nitrate

was freed from acid by fusing the salt before dissolving it. Although the equations already considered do not numerically apply to this cell yet similar phenomena are observed. The safe limits for the deposited weight and the current density were obtained from the preliminary experiments. The deposit, after being thoroughly washed and dried, was weighed. Afterwards it was heated to redness, and, when cool, reweighed. From the difference in the two weighings the correction for the occluded salt was obtained, after Lord Rayleigh and Mrs. Sidgwick.

2. *The Copper Cells.*

These were all of the cylindrical type. The determinations recorded (1) to (4) were the last of the series in which rotating cathodes were used, numbers 1 and 2 being carried out prior to the investigation of the effects of the porous pot. Determinations 5 to 10 are the last six experiments carried out. These were performed after all the preliminary work was completed and form an unbroken set of results, none of the weighings being omitted, except that in brackets in number 5. This is explained in the footnote at the end of the table. All the cells contain porous pots except those of the first two, 3 and 4 have red pots and the rest white ones. In the experiments 1 to 4, as soon as the current was broken rotation was stopped, and the rods were dropped into a similar copper sulphate solution as quickly as possible. The caps were removed, the rods simultaneously rinsed in tap water, and distilled water, allowed to stand in absolute alcohol, wiped with a cloth desiccated, and weighed. It was found that the areas of the cathodes did not vary sufficiently to plot the graph satisfactorily. In the experiments 5 to 10 with stationary cathodes the following modifications were adopted. On a single teak base were arranged four pillars of the same height, carrying the slotted brass plates, which could be swung out of position easily. The whole was heavily coated with shellac varnish. The cells consisted of two battery jars and two crystallising dishes, the latter being raised on teak blocks. White porous pots were added and paraffin wax run in until the appropriate depth was obtained. When the cells were in position, the tops of the porous pots, which reached half a centimetre above the cells, were on the same level. Three of the four cathodes consisted of thin brass tubes, closed at each end with brass discs. They were 12 to 16 cs. long and 2 cs. in diameter. The fourth cathode was a brass rod. The remaining portions of the cells

TABLE I.—CONDITIONS OF EXPERIMENT.

No.	Strength of AgNO_3 solution.	Strength and state of CuSO_4 solution.	Rotation of cathodes. R.P.M.	Current in amps.	Time in mins.	Current density of Cu cathode.		Solution factor $\times 10^5$.	Notes.
						Max.	Min.		
1	2.5—N {	2.5—N + 0.3% acid	} 720	1.0	33	0.10	0.063	—2.0	{ 1 rod stopped resulting in the current density being too high.
2	2.5—N {	2.5—N + 0.3% acid	} 700	1.0	65	0.14	0.067	—6.7	{ Cu reduced in hydrogen.
3	2.5—N {	2.0—N neutral; boiled over Cu on water bath for 2 hours.	{ (a) 0 (b) 600 }	0.6	55	0.045	0.014	{ (a) +0.32 (b) +0.75 }	{ (a) 2 hollow tubes. (b) 3 rods.
4	2.5—N {	1.8—N neutral(?); boiled 3 times as above.	} 830	0.6	54	0.056	0.038	—6.1
5	1.0—N {	2.0—N + 0.1% acid	} Stationary.	0.2	240 + 90	0.018	0.0037	—6.6	{ During absence the circuit was broken after 240 mins., so electrodes stood in electrolyte for another 90 mins.
6	1.0—N	Idem.		0.23	155	0.02	0.0042	{ —1.6 —2.2 }	—1.9 Deposit on copper.
7	2.0—N	Idem.		0.2	270	0.019	0.0038	{ —4.6 —4.2 }	—4.4
8	2.0—N {	2.0—N neutral; water bath for 2 hours.		0.23	275	0.021	0.0042	0	{ Anodes of Kahlbaum's Cu.
9	1.9—N filtered	2.0—N, water bath for 2 hours, +0.1% acid.		0.175	430	0.016	0.0033	—3.6
10	1.9—N filtered	Idem., used 2nd time.		0.3	120	0.025	0.0055	—0.66	{ 1 new porous pot for Ag, well boiled after chemical treatment

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followed the type previously described under the heading "Cylindrical Cell." The copper sulphate solution was poured into the cells to the same distance from the top, so that the deposits were approximately 12, 24, 36 and 48 sq. cs. in area, the rod being used to take the smallest area. In order that the cathodes should be dealt with simultaneously they were treated as follows. After being weighed they were dropped into holes in a stand, the holes being arranged to correspond with the final position of the cathodes in the cells. A carrier was dropped over them and clamped them tightly by means of thumb screws. The electrodes were now transferred to the cells. They were lowered gently into the electrolyte, so that no portion was wetted which did not finally remain immersed, the displaced liquid meanwhile being siphoned into the anode compartment. The carrier was removed and the current started. To wash the deposits the brass plates were swung out of the way, the carrier fixed in position, and the whole quickly transferred to a tank of water placed under the bench. After being thoroughly swilled the tubes were lowered into absolute alcohol contained in four large mouthed bottles, and allowed to stand whilst the carrier was removed. The electrodes were now withdrawn, wiped with a cloth, and placed in desiccators.

In order to detect any sources of error not previously considered the experimental conditions for each determination were varied as much as possible. These are set out in the accompanying Table (I.) for the 10 determinations. It will be seen that the current density varies from 0.14 to 0.0033, and the time of the experiments from 33 to 430 minutes. It may be mentioned that in the earlier series of determinations the strength of the copper sulphate solution varied from 0.5 to 2.5 normal with acid added from 0 to 20 per cent. with concordant results. The solution factor—*i.e.*, the mean loss of copper per unit area of the cathode is shown in column 8. This was obtained by substituting in the equation

$$w = ma + w_0,$$

where w = the weight of the copper deposit,

a = the area of the cathode,

w_0 = the intercept on the axis of zero area—*i.e.*, the value for the corrected weight of copper,

m = the slope of the line, or the solution factor.

This graph was compared to that drawn by eye; the latter was

sometimes difficult to draw where the experimental errors were large, due to the conditions imposed upon the experiment. In the majority of cases the two graphs agreed. In number (8) no satisfactory graph can be drawn, and therefore the solution factor was put equal to zero. The line for the three rods in number (3) as calculated has a small positive slope indicating oxidation. As the points were close together three colleagues were asked to draw the graph by eye. The mean value of the three intercepts was the same as that obtained from the equation. The factor for the tubes is also positive, but differs from that of the rods because they were not washed and dried at the same time. It is interesting to note that in these two cases the electrolyte has been boiled over copper for two hours, rendering it neutral. In determination (4), however, the electrolyte used in (3) has been treated in the same way three times. Its strength has dropped from 2 to 1.8 normal, whilst at the same time it has become acid, having a solution factor -6.1×10^{-5} . This clearly shows the necessity of correcting for solution under experimental conditions. The graphs as calculated and drawn by eye differ somewhat in determinations (6) and (7). The two values of the slopes are given, the calculated value being first. The mean value is used for the correction of the weights.

3. *The following is a typical example.*

Determination 10.

AgNO_3 solution = 1.9N, filtered.

CuSO_4 solution = 2.0N + 0.1 per cent. acid, but
used before in number 9.

Current = 0.3 ampere.

Time = 120 minutes.

Platinum Dishes.

	(1) grs.	(2) grs.
1st weight	32.26799	32.68371
2nd weight	34.70764	35.12310
1st weight of Ag	2.43965	2.43939
Loss on heating to redness	0.00051	0.00037
∴ Weight of AgNO_3 occluded	0.00140	0.00104
∴ Weight of Ag deposited	2.43825	2.43835
Correction for weights	+1	+8
Correction for buoyancy	-6	-6
∴ Corrected weight of Ag	2.43820	2.43837
Mean weight of Ag	2.43829	

TABLE II.—WEIGHINGS.

No.	Corrected weight of silver.	Area of cathode. s.cs.	Weight of copper.	Correction for solution.	Corrected weight of copper.	Atomic weight of copper.	Mean error in atomic weight, due to	
							Cu.	Ag.
1 {	Grs. 2·21697	15·8	Grs. 0·65274	Grs. +0·00032	Grs. 0·65306	Grs. 63·566	} 0·002	} 0·009
	2·21636	13·3	0·65282	0·00027	0·65309	63·569		
	...	10·0	0·65285	0·00020	0·65305	63·565		
		
2 {	...	14·8	1·27259	+0·00099	1·27358	63·565	} 0·002	} 0·000
	4·32294	12·3	1·27259	0·00082	1·27341	63·557		
	4·32294	9·6	1·27287	0·00064	1·27351	63·561		
	...	7·2	1·27298	0·00048	1·27346	63·559		
3 {	...	(a)41·3	0·64419	-0·00013	0·64406	63·559	} 0·004	} 0·010
	2·18669	29·0	0·64415	0·00009	0·64406	63·559		
	2·18601	(b)16·4	0·64428	0·00012	0·64416	63·569		
	...	14·8	0·64418	0·00011	0·64407	63·560		
	...	13·4	0·64425	0·00010	0·64415	63·568		
		
4 {	...	15·9	0·63167	+0·00097	0·63264	63·562	} 0·003	} 0·002
	2·14755	14·3	0·63175	0·00087	0·63262	63·559		
	2·14747	12·5	0·63179	0·00076	0·63255	63·553		
	...	10·7	0·63201	0·00065	0·63266	63·563		
5 {	3·24431	54·5	0·95210	+0·00360	0·95570	63·558	} 0·004	} ...
	*(3·24523)	44·1	0·95289	0·00292	0·95580	63·565		
	(3·24429)	24·7	0·95418	0·00163	0·95581	63·565		
	...	11·0	0·95491	0·00073	0·95564	63·554		
6 {	...	55·0	0·67569	+0·00104	0·67673	63·543	} 0·016	} 0·000
	2·29785	40·0	0·67648	0·00076	0·67724	63·591		
	2·29783	24·8	0·67654	0·00047	0·67701	63·569		
	...	11·2	0·67697	0·00021	0·67718	63·585		
7 {	...	53·2	1·04138	+0·00234	1·04372	63·570	} 0·011	} 0·022
	3·54120	41·8	1·04155	0·00184	1·04339	63·550		
	3·54370	23·0	1·04266	0·00101	1·04367	63·567		
	...	10·7	1·04280	0·00047	1·04327	63·543		
8 {	...	54·7	1·15430	} 0·00000 {	1·15430	63·563	} 0·018	} 0·005
	3·91789	46·0	1·15434		1·15434	63·565		
	3·91852	25·4	1·15362		1·15362	63·525		
	...	11·1	1·15486		1·15486	63·594		
9 {	...	53·1	1·42132	+0·00191	1·42323	63·555	} 0·002	} 0·026
	4·83364	41·8	1·42180	0·00151	1·42331	63·558		
	4·82973	23·9	1·42245	0·00086	1·42331	63·558		
	...	10·8	1·42281	0·00039	1·42320	63·554		
10 {	...	54·8	0·71782	+0·00036	0·71818	63·550	} 0·008	} 0·002
	2·43820	43·6	0·71815	0·00029	0·71844	63·573		
	2·43837	24·8	0·71812	0·00016	0·71828	63·559		
	...	12·0	0·71827	0·00008	0·71835	63·567		

* *Note to No. 5.*—A porous pot, after being allowed to run dry, invariably yields too high a weight for the silver. In this case a new pot was used, which had not been sufficiently boiled after its chemical treatment. If half of the occluded salt be taken as AgCN, then the corrected weight of the Ag deposit 3·24429 is in close agreement with the weight 3·24431. The bracketed weights have not been used in estimating the atomic weight.

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Copper.

Tubes	1.	2.	3.	Rod.
	Grs.	Grs.	Grs.	Grs.
1st weight	41.15292	37.81647	37.26446	32.95332
2nd weight	41.87035	38.53437	37.98221	33.67161
Weight of copper.....	0.71743	0.71790	0.71775	0.71829
Correction for buoyancy...	+30	+28	+27	+7
Correction for weights.....	+9	-3	+10	-9
Corrected weight of Cu (1)	0.71782	0.71815	0.71812	0.71827
Area of cathodes in s.cs. ...	54.8	43.6	24.8	12.0

These weights are plotted as ordinates against the corresponding areas as abscissæ. The line has a slope, which gives for the solution 0.66×10^{-5} grammes per square centimetre, and cuts the axis of zero area at the

corrected weight, 0.71831 gramme.

$$\therefore \text{The atomic weight of Cu} = \frac{0.71831}{2.43829} \times 107.88 \times 2.$$

$$= 63.562.$$

4. Experimental Errors.

In order to determine the extent of the experimental error, each copper weight is corrected for solution by adding to it the product of the area and the constant 0.66×10^{-5} . These corrected values are then compared with the mean silver as follows:—

Area of cathode.	Weight of Cu.	Correction for solution.	Corrected weight of Cu.	Atomic weight of Cu.
S.cs.	Grs.	Grs.	Grs.	
54.8	0.71782	+0.00036	0.71818	63.550
43.6	0.71815	0.00029	0.71844	63.573
24.8	0.71812	0.00016	0.71828	63.559
12.0	0.71827	0.00008	0.71835	63.567

Therefore the mean error in the atomic weight due to Cu = ± 0.008 . Similarly, if the corrected weight of copper 0.71831 is compared with the two silver weights 2.43820, 2.43837, the atomic weights become 63.564 and 63.560. Therefore the mean error in the atomic weight due to Ag = ± 0.002 . Table (II.) shows the weights, &c., for the 10 determinations, the atomic weight being calculated from the mean silver.

In determinations 7 and 9 the values for the atomic weight are vitiated by the large experimental errors of the silver. For this reason they are bracketed in the summary below and are omitted in determining the final value. These errors cannot

be accounted for, the deposits in both cases appearing remarkably good. Smith and Lowry (reference 18) pointed out that a strong solution of AgNO_3 has the power of dissolving AgNO_2 formed during the fusion of the salt. The nitrite may become insoluble in the weaker cathode solution and be deposited with the silver. Corrections made for occluded nitrite instead of the nitrate will raise the value for the atomic weight, but does not decrease the experimental errors. The source of error may be due to the detritus from the porous pots. If such is the case the error would increase with the duration of the experiment, and this seems to be the case.

The errors in the atomic weight due to copper depend largely on the state of the electrolyte.

For neutral solutions the error = 0.01, and greater still when CuO is added.

If 0.1 per cent. acid is added the error = 0.003 when the solution is used once.

If 0.3 per cent. is added the error = 0.002 when the solution is used several times.

Probably the errors are due to the presence of CuO formed by hydrolysis. This can be temporarily removed by filtering through fine Swedish filter paper, and permanently by adding enough acid or by electrolysing the solution. Any of these will impart to the solution the well-known clear sapphire colour.

5. Results.

Summarising the mean values for the 10 determinations we have :—

- (1) 63.567
- (2) 63.560
- (3) 63.563
- (4) 63.559
- (5) 63.560
- (6) 63.572
- (7) [63.558]
- (8) 63.562
- (9) [63.556]
- (10) 63.562

\therefore The atomic weight of copper = 63.563

when $\text{Ag} = 107.88$.

Mean error in the atomic weight = ± 0.003

Mean error due to copper = ± 0.007

Mean error due to silver (omitting 7 and 9) = ± 0.003

Probable error due to weighing = ± 0.003

In these 10 determinations no less than 100 results can be obtained for the atomic weight whose mean value is 63.562.

In conclusion, the author takes this opportunity of putting on record his indebtedness to Dr. J. H. Vincent, in whose laboratory this work was done.

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ABSTRACT.

Four copper cells separating two silver cells were run in series. The areas of the four copper cathodes increased from 10 to 50 s.cs. By plotting the weights of the copper deposits against the corresponding areas of the cathodes, and extrapolating to zero area, the weight of the deposit is corrected for under experimental conditions.

The atomic weight of copper = $\frac{\text{corrected weight of Cu}}{\text{mean weight of Ag}} \times 107.88 \times 2$.

The mean atomic weight for 10 determinations = 63.563, with a mean error of ± 003 . To obtain a uniform coherent deposit of pure metal the following points were considered:—

1. Cylindrical cells with stationary and rotating cathodes were used.

2. The cathode current density must be kept below a certain *limiting* value to prevent the formation of non-coherent deposits, due to secondary deposition. This was found to depend upon the geometry of the cell, the concentration of the electrolyte, the presence of acid and other impurities, the addition of a porous pot and

the rate of revolution of the cathode. Formulæ are given by which the *limiting cathode current density* can be found for all conditions of the cell for Cu, Ag, Au and Zn.

3. To prevent the formation of loose crystalline clusters the current density must also be kept below a certain value depending upon the weight to be deposited. (Formulæ are given.)

DISCUSSION.

Mr. F. E. SMITH congratulated the author on the results of his work. He appeared to have triumphed over many difficulties, and for his particular purpose had converted the copper voltmeter from an instrument of error into an instrument of precision. In future work he hoped that Mr. Shrimpton would avoid the use of common porous pots. They might produce trouble, and were not necessary, as all voltmeter work could now be carried out without the introduction of any medium between anode and cathode. He should be pleased to give Mr. Shrimpton full particulars. The statement respecting deposition of hydrogen was, he thought, incorrect. It was not logical to assume that hydrogen ions were first deposited because of the lack of copper in the neighbourhood of the cathode, and then to state that as soon as they were deposited they went into solution again, and in doing so displaced copper from the solution. With regard to crystalline growths at the bottom of the cathode, he believed these were due to the high current density at the base due to the passage of the current through the liquid descending from the anode. There was also a possibility that the electrolyte was not quite pure. In the silver voltmeter he had obtained striated deposits with impure electrolytes, but not with pure solutions. Mr. Shrimpton also stated that silver deposits from acidified solutions were normal; this was not his experience, nor that of other investigators. He believed the deposit to be less in mass when the electrolyte was acid. A rather important question arose with regard to any Cu_2SO_4 which might be formed in the cathode space. Richards (who also extrapolated to zero area, as Mr. Shrimpton had done) remarked in his Paper: "A value obtained in this way must correspond to a deposit of copper slightly too great; for the mode of correction (*i.e.*, extrapolation to zero area) does not take account of the growing, although slight, presence of cuprous salts." Mr. Shrimpton said the extrapolation to zero area covered this case. The speaker thought the point was one that should be settled. Of course, when one reduced the area the current density at the cathode was increased, and so also was the fall of potential near the cathode; this might have some influence.

Dr. J. H. VINCENT mentioned that after working out the conditions necessary for satisfactory results, the author had performed the 10 determinations quoted in the Paper straight away, no results being rejected. As they all came within 1 part in 6,000 of the mean there appeared to be no doubt that the correct conditions for satisfactory working had been obtained.

Dr. S. W. J. SMITH thought the author's views on secondary deposition could not be correct as they stood, although it was conceivable that sudden variations in the surface concentration of the copper sulphate, due to irregularities of convection, might make it possible for hydrogen to be deposited at one moment and to go into solution again at the next. Non-coherent deposits could be explained without invoking the aid of hydrogen. He thought the author's empirical conclusion, that the limiting current density depends upon the concentration of non-ionised molecules, had some theoretical support.

Mr. SHRIMPTON (communicated remarks): In section II. 2, line 34, as originally written, the author, following Hittorf, described the non-coherent deposit as being due to secondary reaction. As the deposit when the limiting current density was exceeded had the appearance of precipitated copper, it was assumed as a working hypothesis that the hydrogen after being deposited went again into solution, thereby displacing Cu from the unionised CuSO_4 . In the discussion it was pointed out that if the similar deposit of the Ag voltameter be carefully collected and weighed, there is a loss in weight, thus indicating the deposition of H. It seems, therefore, that the above hypothesis is unnecessary, the mere presence of deposited H materially affecting the deposition of the metal. For this reason the text has been altered. As regards the presence of acid in the silver cell, the author meant that the crystalline character of the deposit was not destroyed as in Cu, Zn and Au (the last in aqua regia). In the Ag cylindrical cell acid must not be present because non-coherent needles soon develop at the top of the deposit. Lastly, the author considers that Cu_2SO_4 , if formed, was completely corrected for, since its formation must be a function of the area of the cathode and the duration of the experiment. In the determinations the current density varied from 0.14 to 0.003, and the duration of the experiments from 33 to 430 minutes, yet no difference appears in the values for the atomic weight.

XXXII. *Note on an Improvement in the Einthoven String Galvanometer.* By W. H. APTHORPE.

COMMUNICATED BY MR. R. S. WHIPPLE.

RECEIVED MAY 7, 1914.

SINCE the String galvanometer was introduced by Prof. Einthoven in 1903* it has been found useful for many purposes where an extremely sensitive galvanometer is required; but perhaps it has been put to the greatest use in physiological work in recording small changes in electric potential, due to the movements of the heart. This is done by connecting the patient to the galvanometer terminals and projecting an image of the string by means of an arc lamp and a suitable optical system on to a photographic plate.

As it is necessary to record the cardiac potential from three or more points of the body, and in conjunction with these the phonocardiogram† for a complete examination of the heart, it will at once be appreciated how much more readily the records can be studied, and the exact relation between the phenomena determined if two or more records are obtained simultaneously on the same photographic plate. Up to the present this has only been possible by using two separate string galvanometers,‡ each having its own optical system and lamp for illuminating the string, the optical systems being so adjusted that the projected images fall side by side on the camera.

It has been possible in this way to obtain good results after expending a considerable amount of time and patience in making the necessary adjustments.

In December, 1912, Dr. Avery Newton suggested that the same result could be obtained more easily by placing two strings in the same magnetic and optical field, so that the two images were projected side by side on to the photographic

* "Annalen der Physik," 1903, XII., 1,059.

† A record of the sound of the heart beating which is produced by converting it into an electric current by means of a stethoscope, a microphone and transformer.

‡ It should be mentioned that an oscillograph, with two or more moving systems in the field of an electromagnet, has been made by Messrs. Bock and Thoma of Munich, and that it has been used for taking simultaneous electrocardiograms.

plate. This, however, was found to be practically impossible, owing to the closeness with which the two strings had to be mounted (0.04 mm.) in order that the two images should be sufficiently close together under a magnification of 600. This large magnification is found necessary in cardiographic work.

In order to overcome this difficulty the following device was

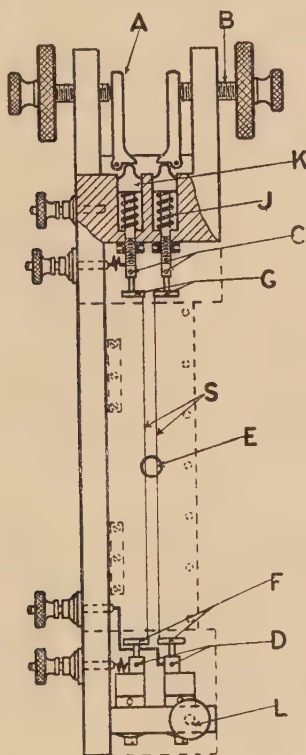


FIG. 1.

recently constructed, which has been found to give very satisfactory results.

A special holder, as shown diagrammatically in Fig. 1, has been made for holding two strings parallel and in a plane at right angles to the magnetic field. The strings *S*, which are made of silvered glass fibres, are soldered to T-shaped brass ends *F* and *G*, which are held in position by set screws at *C* and *D*.

Provision is made for adjusting the tension by means of the screws and bell crank levers A and B. By screwing in the screws the levers depress the plungers J and K to which the strings are attached to one end, and so decrease the tension and thus alter the sensitivity.

A further adjustment was provided at L for moving one string in relation to the other, so as to bring the parts of the strings under observation into the same focal plane. The strings are placed at a distance of about 0.5 mm. apart, which was found to be convenient for manipulation and which allows both of them to come well within the field of a 16 mm. Zeiss objective.

The moving systems are entirely enclosed in a thin metal case (shown dotted) and a small mica window E allows the movements to be observed at the centre.*

As it is desirable to record the deflections of the strings on a photographic plate $3\frac{1}{4}$ in. (82 mm.) wide, it is necessary to reduce the distance between the projected images. This is done most conveniently by deflecting the projected beam by means of an achromatic prism of about 15 deg. as shown at M and N Fig. 2.

The final adjustment of the distance between the images is obtained by rotating the prisms through a small angle about their common vertical axis O.

By means of this device not only is the manipulation simplified, but as the same time marker is used, any slight inaccuracy in timing will affect both records to the same extent, and can usually be neglected. Both strings are influenced equally by any change in the magnetic field.

The actual method of mounting the fibre case, the micrometer heads for altering the tension of the fibres, and the arrangement for supporting the prism are shown clearly in Fig. 3.

Fig. 4 is part of a record of an electrocardiogram and a phonocardiogram obtained simultaneously from a patient by means of this apparatus.

The upper record is the electrocardiogram obtained from

* The closed metal case for holding the moving system of a string galvanometer was originally designed by Mr. W. Duddell in 1905, for the Cambridge Scientific Instrument Co., Ltd. It is by the courtesy of the company that I am able to describe this improvement which has been developed in their workshops. I may also mention that patents have been applied for to protect the invention.

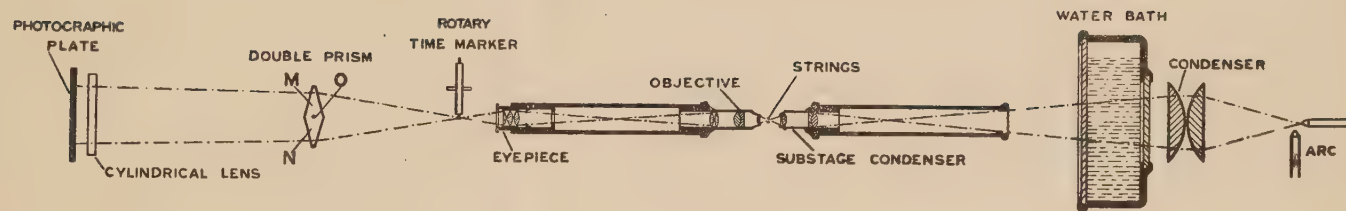


FIG. 2.

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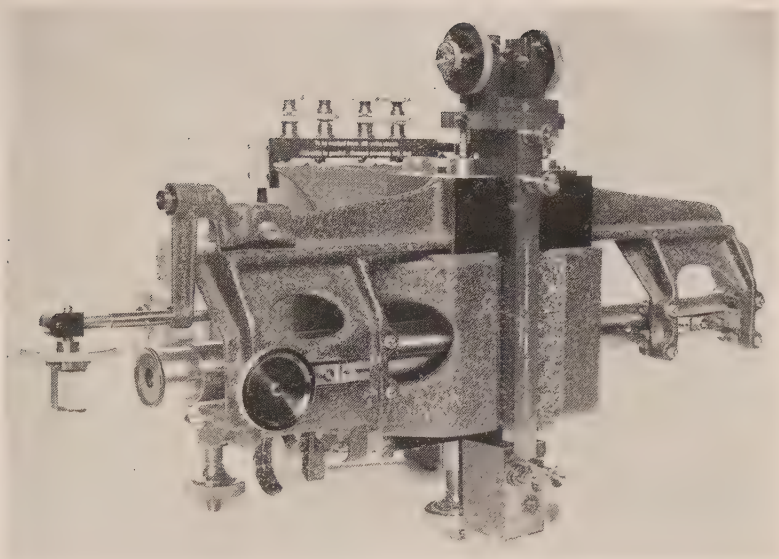


FIG. 3.

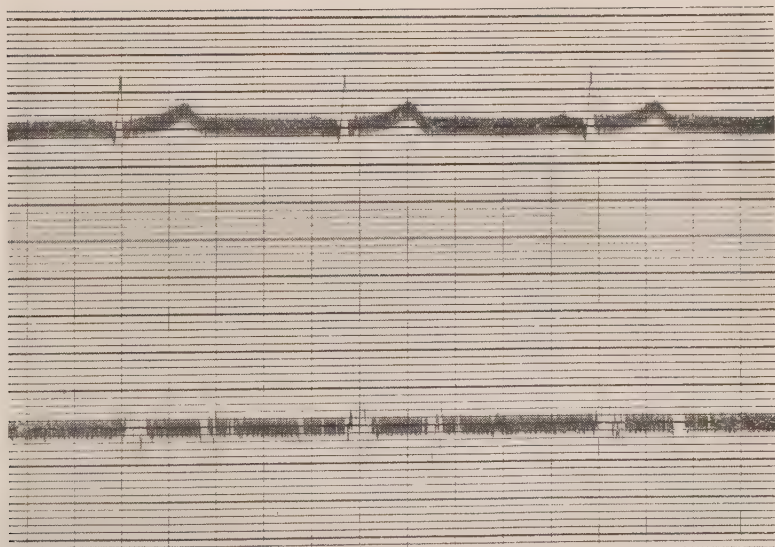


FIG. 4.

To face page 316.]



lead I.* of a normal patient, the lower record being the phonocardiogram obtained simultaneously with the electrocardiogram.

The horizontal lines are produced by lines engraved on the cylindrical lens of the camera, each division representing 0.0001 volt. The vertical lines are time intervals of one-fifth of a second.

Although this apparatus was designed for electrocardiographic work it appears to the author that it might be employed with advantage for other purposes, such as recording two radio-telegraphic signals of different wave length simultaneously by means of a camera in which a band of photographic paper is made to move continuously.

* Following the nomenclature of Einthoven, the different circuits through the patient and the galvanometer, which are generally used, are called "Leads," and are numbered as follows :—

Lead I., that obtained with the horizontal connection (right arm, left arm).

Lead II., that obtained with the oblique connection (right arm, left leg).

Lead III., that obtained with the vertical connection (left arm, left leg).

XXXIII. *On Atmospheric Refraction and its Bearing on the Transmission of Electromagnetic Waves Round the Earth's Surface.* By J. A. FLEMING, M.A., D.Sc., F.R.S.

RECEIVED MAY 28, 1914.

APART altogether from true diffraction or from any reflective or refractive effects due to atmospheric ionisation the normal refraction of the atmosphere due to the decreasing density of the constituent gases as we rise above the earth's surface exercises a bending effect upon rays of light, and therefore also upon electric radiation sent out from any point in various directions. If a ray of light is sent out from a source on the earth's surface, starting in a direction tangential to the terrestrial spheroid, it follows a curved path of varying curvature with concavity directed towards the earth, as it passes outwards through the atmosphere. This curvature is such that, very roughly speaking, a tangent to the ray is bent through an angle of about half a degree in its course.

Conversely, the atmospheric refraction bends incoming rays from stars or celestial bodies so that their apparent zenith distances are less than the true ones.

It becomes, therefore, an interesting matter to inquire under what conditions this ordinary atmospheric refraction taken alone could operate to carry a ray, starting from any point on the earth's surface in a tangential direction, round the earth parallel to its surface. The question is not by any means a new one, and has already been considered in connection with some astronomical problems.* It has, however, become increasingly clear of late years that the full explanation of the phenomena of long-distance radio-telegraphy, such, for instance, as the transmission of wireless signals a quarter of the way round the earth from Ireland to Argentina, is only to be found in atmospheric action on the transmitted waves. The most complete mathematical investigations, such as those of Prof. H. M. Macdonald,† March and Rybczynski,‡ show that it is not possible for true diffraction to contribute more than a certain

* See Dr. August Schmidt, "Die Strahlenbrechung auf die Sonne," Stuttgart, 1891. Also Miss Agnes Clerke, "Problems in Astrophysics," p. 165.

† "The Transmission of Electric Waves around the Earth's Surface," "Proc." Roy. Soc., A., Vol. XC., p. 50, 1914.

‡ "Ann der Phys," Vols. XXXVII. and XLI.

fraction of the total effect at the distant receiving station. On the other hand, whilst there are indications that some of the effect is propagated along or through the earth's crust and follows round any irregularities in it, it is also clear that the major part cannot arrive in this manner or it would not be so dependent upon changes in the state of the atmosphere as regards solar illumination, as we find it to be.

We are, therefore, thrown back upon the consideration of the question of the relation of the atmosphere to electric wave propagation through it. The extraordinary vagaries of long electric wave transmission and the manner in which our power of transmitting such waves varies from hour to hour and day to day through the year points to some continually fluctuating atmospheric condition as a principal factor in it. Such a condition is found in the ever-varying ionisation of the atmosphere under solar light.

Dr. W. H. Eccles has provided the foundation for a valid theory of ionic refraction and an explanation of the manner in which ionisation of the upper air can cause an electric ray to be bent round so as to follow more or less the earth's curvature in its path.*

Also he has considered the influence of what may be called the normal refraction of the atmosphere, and suggested some modes of variation of refractive index with height above the earth's surface which would tend to fulfil the requirements of circular propagation of the ray round the earth.†

The whole question of long-distance radiotelegraphy is, therefore, intimately bound up with fundamental questions of meteorology and atmospheric composition, and it may be well to refer briefly to our knowledge on the subject. A very valuable summary of information on it from the point of view of chemistry and spectroscopy was given by Sir James Dewar in his Presidential Address to the British Association at Belfast in 1902, and also in a Friday Evening Discourse at the Royal Institution on April 11, 1902, on "Problems of the Atmosphere." (See "Proc." Roy. Inst., Vol. XVII., p. 223.)

He showed that, broadly speaking, the atmosphere may be divided into two regions. First, a lower region in which winds

* See Dr. W. H. Eccles, "Proc." Roy. Soc., A, Vol. LXXXVII., p. 79, June, 1912.

† See "British Association Report," Birmingham, 1913. "Proc." Sec. G., and "The Electrician," Sept. 19th, 1913. "Atmospheric Refraction in Wireless Telegraphy."

and convective motions keep the constituents so well mixed that although there is a gradual decrease of density as we ascend the chemical composition is not very much changed, except in the percentage of water vapour and carbon dioxide.

This convective region extends to a height of 10 or 20 miles, more or less.

Above that height we pass into a second region in which convection ceases to operate and differences of density in the gaseous constituents and other more obscure actions cause a separation to take place. The denser gases oxygen and nitrogen gradually disappear at some height dependent on the temperature gradient, and above this the atmosphere consists practically of hydrogen and helium, with possibly the other more volatile atmospheric gases—neon, krypton, &c.—present in some degree.

Sir James Dewar gave at his Royal Institution lecture above mentioned diagrams representing the atmospheric composition at various heights on certain assumed temperature gradients. He stated that on any permissible temperature-gradient hypothesis the atmosphere above 60 miles or so would be substantially composed of hydrogen and helium.

If we assume a homogeneous atmosphere and neglect the variation of gravity with height, and assume temperature to be constant or to decrease upwards according to some straight line law, it is a comparatively easy matter to calculate the density at any height in a column of gas in equilibrium under the forces of gravity and its own elasticity. Thus, if p_0 and q_0 are the pressure and density of the gas at the earth's surface, and p and q the same at any height h above it, and if T is the absolute temperature, g the acceleration of gravity and G the gas constant, then we have

$$p = GTq. \quad \dots \dots \dots (1)$$

If we consider a horizontal slice of thickness, δh , of a vertical column of the gas of unit cross-section, then the equation of equilibrium of the element is

$$-\frac{dp}{dh}\delta h = gq\delta h, \quad \dots \dots \dots (2)$$

or

$$\frac{dq}{q} = -\frac{g}{GT}dh. \quad \dots \dots \dots (3)$$

If R is the mean radius of the earth, and if the mean value of gravity at the surface is 980, then $g = 980R^2/(R+h)^2$ nearly.

Since the variation of gravity is only about 5 per cent. in a height of 100 miles we make no great error in considering it as a constant. There is more difficulty in finding a function of h to express the temperature. One of the results brought to light by modern observations with sounding balloons is that the atmosphere may be divided into two regions in which the distribution of temperature is very different. In the lower region there is a continual fall in temperature up to a height of about 10 km., or 6 miles or so, at the rate of about 6°C . per kilometre. Above this level the temperature becomes constant for a certain unascertainable range. Beyond that it is reasonable to suppose there is a second gradient of temperature falling towards the temperature of space. The lower region in which the first temperature gradient takes place, which is more or less identical with the region in which convection and winds keep the atmospheric gases well mixed, is now called the *troposphere*. The region of more constant temperature is called the *stratosphere*. The height at which the stratosphere is reached depends on the latitude, being lower in polar than equatorial regions.

If we consider the temperature and gravity constant at various heights the integral of equation (3) is

$$q = q_0 e^{-\frac{g}{GT}h}, \quad \dots \dots \dots (4)$$

showing that the density decreases in accordance with an exponential law.

If we assume a linear temperature gradient of $\beta^{\circ}\text{C}$. per kilometre, so that T is a function of the height h of the form,

$$T = T_0 \left(1 - \frac{\beta}{T_0} h \right), \quad \dots \dots \dots (5)$$

where $\beta = 6$ nearly, and T_0 is the absolute temperature at the earth's surface, then equation (3) takes the form

$$\begin{aligned} \frac{dq}{q} &= -\frac{g}{GT_0} \frac{dh}{(1 - \beta h/T_0)} \\ &= \frac{g}{G\beta} \frac{d(1 - \beta h/T_0)}{(1 - \beta h/T_0)}. \quad \dots \dots \dots (6) \end{aligned}$$

The integral of this is

$$q = q_0 \left(1 - \frac{\beta}{T_0} h \right)^{g/G\beta}, \quad \dots \dots \dots (7)$$

subject to the condition that h must not be greater than T_0/β .

If β is taken as the temperature fall in absolute degrees per kilometre, then since the dimensions of g/G are the inverse of a length, and since $1 \text{ km.} = 10^5 \text{ cm.}$, we have to multiply $g/G\beta$ by 10^5 if we take the unit of length as the kilometre. Again, if we take gravity as 980 and the absolute density of air as $1/800$ at a pressure of 10^6 dynes per square centimetre, then we have

$$\frac{g}{G\beta} = \frac{980 \times T_0 \times 10^5}{800 \times \beta \times 10^6} = \frac{49}{400} \frac{T_0}{\beta},$$

the kilometre being the unit of length.

If we take the average temperature of the air at the earth's surface to be $300^\circ \text{abs.} = T_0$, and if $\beta = 6^\circ$ per kilometre then $g/G\beta = 49/8 = 6.125$. This is rather a high value for T_0 , but we take 300 to get a round number.

We have then

$$q = q_0 \left(1 - \frac{h}{50}\right)^{6.125} \dots \dots \dots (7a)$$

This last expression, which takes account of the temperature gradient, is really more simple than the exponential expression (4), but (7) is limited in range to T_0/β km., at which $q=0$, whereas (4) gives $q=0$ at an infinite distance. It is clear, however, that β is not constant throughout the whole height of the atmosphere.

Throughout the region of the troposphere the formula (7) gives a ratio for q/q_0 numerically less than that given by (4). Thus, if $h=5$ km., and if T_0 and β have the values given above, we have $g/GT=0.1225$, and (4) gives us

$$\frac{q_0}{q} = e^{0.6125} = 1.845,$$

whereas (7a) gives

$$\frac{q_0}{q} = \left(\frac{10}{9}\right)^{6.125} = 1.898.$$

If $h=10$ km., then (4) gives the ratio $q_0/q=3.404$, whilst (7a) gives it 3.920.

This is as it should be, because lowering the temperature causes the density to degrade more rapidly.

We can next consider the atmospheric condition above the so-called convective region. In the latter the ingredients of the atmosphere are kept so well mixed by winds that the

relative proportions will not be altered with height, although the absolute density will be reduced.

If, then, we insert in (7a) the value $q_0 = 0.001293$, this being the density of air at the earth's surface, it will give us the density q at any height below 10 km. fairly accurately.

Above the convective region Dalton's law will begin to assert itself, and the constituent gases will attenuate independently, each gas attenuating in density according to the law as expressed by equations (4) or (7). Since the gas constant varies inversely as the density of the gas it follows that the denser gases fade away soonest.

Thus, if we take the average temperature gradient throughout a range of 150 km. to be 2° per kilometre then in formula (7) we have to put $\beta = 2$. Also $g/G\beta = q_0 14,700$; if we take the surface temperature as 300° abs. Therefore, we have

$$\frac{q}{q_0} = \left(1 - \frac{h}{150}\right)^{q_0 14,700},$$

or
$$\log_{10} \frac{q}{q_0} = q_0 14,700 \log_{10} \left(1 - \frac{h}{150}\right).$$

Consider the atmospheric composition at a height of 100 km.

Then, $1 - \frac{100}{150} = \frac{1}{3}$. But $\log_{10} 1 = 0$, and $\log_{10} 3 = 0.47712$.

Hence $\log_{10} \frac{q_0}{q} = 7013.7 q_0$.

Table I. below gives the absolute densities of various atmospheric gases at 10^6 dynes per square centimetre pressure and 0°C .

TABLE I.

Gas.	Relative density.	Absolute density.
Hydrogen	1	0.00008837
Helium.....	2	0.00017674
Neon	10	0.0008837
Nitrogen	14	0.0012393
Oxygen	16	0.0014107
Argon.....	20	0.0017674
Krypton	40.88	0.0036125
Xenon	64	0.0056428
Air	—	0.001293

If we multiply these absolute densities by 7013.7 and find the anti-logarithms we have the values of q_0/q , that is, the ratio

of the density at the surface to the density at a height h km., as follows :—

TABLE II.

Gas	q_0/q at 100 km.
Hydrogen	4.167
Helium.....	17.36
Neon	1,570,000
Neon	475,500,000
Nitrogen	8,257,000,000
Oxygen	

The above figures show without further explanation that at a height of 100 km. or 60 miles the atmosphere consists, as stated by Sir James Dewar, substantially of hydrogen and helium. At 50 km. height there is, however, a very large proportion of nitrogen and some oxygen.

Sir James Dewar recalls an observation of Pickering on the spectrum of a meteorite just entering the earth's atmosphere, which showed strongly the lines of hydrogen and helium, as confirmatory of the above conclusions as to the composition of the atmosphere at great heights.

These two formulæ (4) and (7) are therefore practically in agreement with the formulæ of Heinrich and Ferrel, from which the charts were prepared, given in Sir James Dewar's Royal Institution Discourse (*loc. cit.*). By the summation of expressions such as (7), with various values for β , we could represent the change in density due to any assigned mode of temperature variation.

We are now in a position to consider the refractive effects of the atmosphere at various heights. In gases the refractivity ($\mu-1$) is connected with the density q with fair accuracy by Gladstone and Dale's law $(\mu-1)/q=A=a$ constant. The constant A may be called Gladstone and Dale's constant. Its value for various gases are given in Table III. We can then calculate the curvature of a ray passing through the atmosphere as follows :—

The atmosphere may approximately be considered as formed of concentric layers of gas having refractive indices which decrease as the altitude increases. Let us take the centre of the earth as origin and consider the curved path of any ray travelling to or from the earth (*see* Fig. 1). Then if a tangent is drawn at any point P to this curve, and if l is the length of the perpendicular let fall from the origin on the tangent, and μ the refractive index at the point of contact of the tangent, with the curve, it can easily be shown that all along the path of the ray the product $l\mu$ is constant (*see* Parkinson's "Optics," p. 109).

Again, if r is the radius vector to that point, and if ρ is the radius of curvature at the same point, it is easily shown that $\rho = r dr/dl$.

Also by Gladstone and Dale's law, the quotient $(\mu - 1)/q = A$, where q is the density at that point, will be a constant different for each gas. Taking the expression (4) to represent the

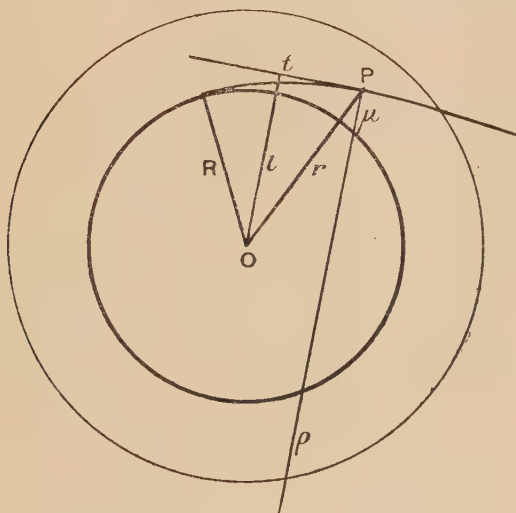


FIG. 1.

variation of density with height, we have the following three equations* :—

$$l\mu = C, \quad (8)$$

$$\frac{\mu - 1}{A} = q = q_0 e^{-a(r-R)}, \quad (9)$$

$$\rho = r \frac{dr}{dl}, \quad (10)$$

where C and A are constants and r is the radius vector from the centre of the earth to any point on the ray at which the refractive index of the air is μ , whilst l is the length of the perpendicular from the centre of the earth on the tangent to the ray at that point. Also ρ is the radius of curvature of the ray at that point, and R is the mean radius of the earth and

* Dr. Eccles starts with three similar equations his investigation on "Atmospheric Refraction in Wireless Telegraphy" (*loc. cit.*, Note, p. 319).

$(r-R)$ is the height above the earth's surface. The symbol α stands for g/GT , where g is the acceleration of gravity, G is the gas constant, and T the absolute temperature.

From (10) we have

$$\varrho = r \frac{dr}{dl} = r \frac{dr}{d\mu} \frac{d\mu}{dl}, \quad \dots \quad (11)$$

and from (8)

$$\frac{d\mu}{dl} = -\frac{C}{l^2} = -\frac{\mu^2}{C}. \quad \dots \quad (12)$$

Hence

$$\varrho = -\frac{r}{d\mu/dr} \frac{\mu^2}{C}. \quad \dots \quad (13)$$

But by (9) we have

$$\mu = 1 + Aq_0 e^{-\alpha(r-R)}. \quad \dots \quad (14)$$

Therefore

$$\frac{d\mu}{dr} = -\alpha Aq_0 e^{-\alpha(r-R)}. \quad \dots \quad (15)$$

Hence substituting (15) in (13) we have

$$\varrho = \frac{r e^{\alpha(r-R)}}{C \alpha A q_0} (1 + Aq_0 e^{-\alpha(r-R)})^2, \quad \dots \quad (16)$$

or by (9)

$$\varrho = \frac{r}{C \alpha} \frac{\mu^2}{\mu-1} = \frac{r}{l \alpha} \frac{\mu}{\mu-1}. \quad \dots \quad (17)$$

Now

$$\alpha = \frac{g}{GT} = \frac{g q_0}{p_0} = \frac{980 q_0}{10^6}.$$

Also q_0 the density of the air at the earth's surface under a pressure of 10^6 dynes per square centimetre is nearly equal to $1/800$. Hence, $\alpha = 98/(8 \times 10^7)$.

The dimensions of α are inversely as a length. Hence, if we take the centimetre as unit of length $1/\alpha = 8 \times 10^5$ nearly. If, however, we reckon lengths or heights in kilometres then the numerical value of $\alpha = 1/8$ nearly.

Accordingly the following simple expression gives the radius of curvature of the ray

$$\varrho = \frac{8r}{l} \frac{\mu}{\mu-1}. \quad \dots \quad (18)$$

Consider, then, a ray starting horizontally from a point on the earth's surface. We have at that point $r=l=R$ =the earth's

radius, and $\mu=1.000294$. Hence, $\mu/(\mu-1)=10,000/3$ nearly, and the radius of curvature of the ray at starting $=80,000/3=26,666$ km., or 16,000 miles, or about four times the radius of curvature of the earth.

As the point considered is taken farther from the source the radius of curvature of the ray at that point increases. For we have

$$\varrho = \frac{r}{l\alpha} \frac{\mu}{\mu-1} = \frac{r\mu}{l\alpha Aq}. \quad \dots \dots (19)$$

But μ is very nearly unity, and A and α are particular constants for each gas, whilst l is never very much greater than r , and the density q continually diminishes with increasing altitude.

Hence, ϱ rapidly increases with increase of distance of the point on the ray considered, as we take it farther from the source on the earth's surface.

Since α is equal to gq_0/p_0 , it is clear, if lengths are measured in kilometres, that $\alpha=98q_0$. Also $\mu_0-1=Aq_0$. Accordingly, taking the source at the earth's surface, where l and r are equal, and considering the ray emitted tangentially by the earth's surface, we have

$$\varrho = \frac{\mu_0}{98A} \frac{1}{q_0^2}. \quad \dots \dots (19a)$$

But μ_0 for all gases is very nearly unity, and A has values for various gases as in Table III. below.

The constant A may be called the Gladstone and Dale constant, and q_0 is the density of the gas at 0°C . and 10^6 dynes per square centimetre pressure.

Gas.	TABLE III. Refractive index μ_0 at 0°C . and 10^6 dynes/cm. ²		$A = \frac{\mu_0 - 1}{q_0}$.
Hydrogen	1.000138	1.56
Helium.....	1.000035	0.197
Neon	1.0000687	0.0777
Nitrogen	1.000300	0.242
Oxygen	1.000272	0.193
Argon.....	1.000285	0.161
Krypton	1.000424	0.117
Xenon	1.000693	0.123
Air	1.000293	0.227

In the formula (19a), if we substitute the values for air, viz., $\mu_0=1.000293$, $A=0.227$ and $q_0=0.001293$ we find $\varrho=27,000$ km. as before, or 4.1 times the earth's radius.

Again, if we suppose the atmosphere consisted wholly of

hydrogen, then inserting in formula (19a) the values $\mu_0 = 1.000138$, $A = 1.56$ and $q_0 = 0.00008837$, we find $\rho = 880,000$ km. or 136 times the earth's radius.

If, however, the atmosphere consisted wholly of krypton, for which $\mu_0 = 1.000424$, $A = 0.117$ and $q_0 = 0.0036125$, we find $\rho = 6,682$ km., or about the same as the earth's mean radius.

Finally, if the atmosphere consisted wholly of xenon, for which $\mu_0 = 1.000693$, $A = 0.123$, $q_0 = 0.005643$, then we find $\rho = 2,768$ km. which is much less than the earth's radius.

On the assumption, therefore, above made as to uniform temperature we find the following remarkable result.

If the earth's atmosphere consisted wholly of krypton a ray of light sent out tangentially to the earth's surface would be refracted round the earth parallel to the surface, and never escape at all from the atmosphere. If, therefore, the atmosphere consisted entirely of krypton, wireless telegraphy right round the earth might be easily possible in consequence of this circular refraction of a tangentially emitted ray.

This curious result can be confirmed in another way. If we consider a plane wave travelling round the earth with wave front always vertical or in a radial direction then the condition for such a wave progress is that the velocity of the wave v at a height h above the earth's surface must be to its velocity v_0 at the surface as $h+R$ is to R , where R is the earth's mean radius. If μ and μ_0 are the refractive indices at these places then for ray transmission round the earth parallel to the surface we must have

$$\frac{h+R}{R} = \frac{v}{v_0} = \frac{\mu_0}{\mu}. \quad (20)$$

But $\mu - 1 = Aq$, hence

$$\frac{h}{R} + 1 = \frac{Aq_0 + 1}{Aq + 1} \quad (21)$$

or

$$\frac{h}{R} = \frac{A(q_0 - q)}{Aq + 1} \quad (22)$$

Differentiate this last equation with regard to h , and we have

$$Aq + 1 + Ah \frac{dq}{dh} = -AR \frac{dq}{dh} \quad (23)$$

or

$$-\frac{dq}{dh} = \frac{q + A^{-1}}{h + R} \quad (24)$$

Hence at the surface of the earth where $h=0$ and $q=q_0$ we have

$$-\frac{dq}{dh} = \frac{q+A^{-1}}{R}. \quad (25)$$

But now by equation (4) we have

$$q = q_0 e^{-\frac{g}{GT}h}. \quad (26)$$

Hence, differentiating (26) with regard to h , we have

$$-\frac{dq}{dh} = \frac{gq}{GT}. \quad (27)$$

Therefore equating (25) and (27)

$$\frac{q+A^{-1}}{R} = \frac{gq}{GT}. \quad (28)$$

Hence

$$GTq + \frac{GT}{A} = gRq. \quad (29)$$

But

$$\mu - 1 = Aq.$$

Therefore,

$$\mu - 1 = \frac{1}{\frac{Rg}{GT} - 1}. \quad (30)$$

But $g/GT = \alpha = 98A$, as already shown, and since Rg/GT is always very much greater than unity, the formula (30) is equivalent to

$$R = \frac{1}{98Aq^2}. \quad (31)$$

This is practically identical with formula (19a), when $\varrho = R$. Hence (31) gives us the value of q required for circular refraction. Accordingly if the terrestrial atmosphere consisted wholly of a gas having a density about twice that of air, but the same Gladstone and Dale constant A , then the radius of curvature at starting of a ray of light sent out tangentially to the earth's surface would be identical with the curvature of the earth, and the ray would follow round the earth and never leave it.

It remains to consider the modification which will be introduced into the expression (19a) for the radius of curvature of the ray if we take into account the temperature gradient.

If in place of the expression (4) for the density at various

heights, which neglects temperature gradient, we use the expression (7) which assumes a uniform temperature gradient, and employ (7) in place of (4) in obtaining the formula for the radius of the curvature of the ray it is quite easy to show that we obtain the result

$$\varrho = \frac{r}{\tan \alpha} \frac{\mu}{\mu - 1} \left(1 - \frac{\beta}{T_0} r - R \right), \quad . \quad . \quad . \quad (32)$$

where the letters have the same signification as in equations (8) to (19). Hence when $h = r - R = 0$ the last expression becomes identical with (19a) and also with (31).

The same expression is therefore obtained in three different ways for the radius of curvature of a horizontally emitted ray at the point of emission.

There appears to be a considerable amount of evidence that many of the spectral lines in the aurora spectrum are due to the more volatile constituents in the air. The question is discussed at length in the Presidential Address and lecture of Sir James Dewar, already mentioned. If any auroral lines due to electric discharges at a height above 40 km. or 50 km. are lines of krypton or xenon then the question presents itself: How do these dense rare gases rise to such heights? A very similar anomaly is seen in the solar atmosphere. The outer portions of the sun's atmosphere, like that of the earth, are composed principally of hydrogen and helium. But calcium, which has an atomic weight 40 is also present in the chromosphere, and no answer has yet been given to the question, by what actions an element of this molecular weight can rise so high. If one may be so bold as to hazard a rash suggestion, perhaps the neon and krypton are manufactured *in situ* by atmospheric electric discharges passing through the rarefied hydrogen, in the manner suggested by recent researches of Profs. Collie and Patterson.

Another question which suggests itself for consideration is, whether the ionisation of the upper regions of the atmosphere, upon which the actual bending of radio-telegraphic waves partly depends, is more easily effected because of the presence there of these non-valent gases neon, helium, &c. The high conductivity and small dielectric strength of these gases must be connected with ease of ionisation. It might be worth while to test the facility for ionisation of different gases under ultra-violet light.

In conclusion, the effect of a variation in the earth's diameter

may be pointed out. Referring to equation (30) since Rg/GT is always large compared with unity, the equation can be written

$$Rg = \frac{GT}{\mu_0 - 1} = \frac{GT}{Aq_0} \quad \dots \dots \dots (33)$$

But at 0°C ., $GT = 10^6/q_0$. Hence for the same temperature and gas the product Rgq_0^2 is constant.

Now gravity at the surface of a planet varies as the radius, if the mean density of the planet remains constant. Hence for the same chemical constitution and the same pressure and temperature of the atmosphere the conditions for the circular refraction of a tangential ray or its following closely round the surface will be complied with if the radius of the planet varies inversely as the surface density of the atmosphere.

If the Gladstone and Dale constant A had a constant value of 0.25 then on our earth, it would be necessary for the atmosphere to have double its present density to fulfil the condition for circular refraction. Therefore, for the same density as at present it would be sufficient if the earth had double its present radius also to fulfil it.

This shows that on a large planet such as Jupiter, having also a dense atmosphere, no ray of light emitted at or near the horizontal direction at any point could escape from the atmosphere. Also it follows that, except in so far as distant vision was hindered by absorption of light, it would be possible to see objects on the surface at a much greater angular distance than is possible on our earth.

These conclusions have merely a theoretical interest, but they show how closely the possible range of long-distance radiotelegraphy is connected both with the size of the planet and the nature of the atmosphere in which it is conducted.

Having regard to the great variations which exist in planetary atmospheres it is quite possible our earth is unique in this respect, as in many others, in being perhaps the only planet on which long-distance wireless telegraphy is possible.

ABSTRACT.

In this Paper the author considers the conditions under which true atmospheric refraction would be sufficient to carry a ray of light or electromagnetic radiation sent out horizontally from any point on the earth's surface round the earth parallel to its surface. It is now generally agreed that pure diffraction is insufficient to account for all the phenomena of long-distance wireless telegraphy, but that we

have to postulate some action of the atmosphere which tends to curve the radiation round the earth. The theory of ionic refraction has been put forward, which is based on the theoretical conclusion that in ionised air the velocity of long electric waves is increased. We know as a matter of fact that the atmosphere decreases in density as we rise upwards, and this alone produces a decrease of refractive index and an increase in velocity.

The first part of the Paper is concerned with the deduction of formulæ expressing this variation of density with heights taking into account as far as possible the known temperature variation with increase of height. It is shown that at a height of 100 km. the terrestrial atmosphere must consist substantially of hydrogen and helium. An expression is then obtained for the radius of curvature at any point of a ray of light sent out horizontally from the earth's surface, and it is shown that this radius at the starting point is given by the formula $\rho = \mu_0 / (98Aq_0^2)$, where μ_0 and q_0 are the refractive index and density at the surface of the earth, and A is the Gladstone and Dale constant for the gas which forms the atmosphere. From known values for various gases it is shown that for air ρ is four times the earth's radius, for hydrogen 136 times, and for krypton equal to the earth's radius. Accordingly if the terrestrial atmosphere consisted wholly of krypton a ray sent out horizontally would be refracted round the earth, and in such an atmosphere wireless telegraphy to the Antipodes would be possible. The above formulæ are deduced in three ways. It is also shown that for the same atmospheric density and constant A this circular refraction would result if the earth were twice its present diameter.

The question of atmospheric composition is then considered in the light of what is known about the auroral spectrum. The suggestion is made that perhaps the non-valent gases neon and krypton are manufactured at great atmospheric heights by electric discharges occurring in the rarefied hydrogen atmosphere. Also that by their ease of ionisation they contribute to produce the ionised layer demanded by the theories of Heaviside and Eccles to account for the actual achievements of long-distance wireless telegraphy.

Finally, it is suggested that our earth is perhaps unique in being the only planet on which such long-distance radiotelegraphy is possible.

DISCUSSION.

Mr. DUDELL considered it was very difficult to follow what was going on in long-distance transmission. One difficulty was that it seemed probable that the heavier gases were absent from the upper atmosphere, yet we had to assume their presence either to get the refraction effect or the Eccles effect.

Dr. C. CHREE thought Prof. Fleming's Paper emphasised the importance of the field common to meteorology and wireless telegraphy. With regard to the relations between temperature, pressure and altitude in the atmosphere, aqueous vapour was an element which meteorologists had to take serious account of; its variability exercised a great influence on the results within 2 km. or 3 km. of the ground. With respect to the constitution of the upper atmosphere, more or less successful attempts had been made of late years to obtain samples by means of apparatus sent up by pilot balloons. On the theoretical side, there were Papers more

recent than the pioneer one by Sir James Dewar quoted by Dr. Fleming. In particular he would call attention to two Papers by Wegener in the "Physikalische Zeitschrift" for 1911. A fundamental point was whether the auroral spectrum did or did not connote a gas different from all hitherto isolated at the earth's surface. Wegener thought it did, and believed the unknown gas thus indicated to be a very light one, which at great heights was even more important than hydrogen. The fact that Prof. Störmer's recent photographic determination of auroral heights had in some cases supplied heights well over 300 km. was evidence that an atmosphere of some kind extended to a very great height. The fact emphasised by Dr. Fleming that wireless results by day and night differed markedly certainly seemed to support strongly his contention that the upper atmosphere played a most important part in the phenomenon. A similar conclusion had been drawn in the case of the ordinary diurnal variation of the elements of terrestrial magnetism, where normally changes were much larger by day than by night, and much larger in summer than winter. It had been found, however, that while the difference between midsummer and midwinter existed in high latitudes, the difference between night and day was there much reduced, even at the equinoxes. Systematic wireless observations in the Arctic or Antarctic might throw a great deal of light upon the whole question.

Prof. G. W. O. HOWE congratulated Prof. Fleming on his interesting and suggestive Paper, and pointed out that although the upper atmosphere undoubtedly had a profound effect on the transmission of electromagnetic waves over long distances, they were still uncertain to what extent long-distance radiotelegraphy would be possible without any assistance from the upper atmosphere. As successive mathematicians attacked the problem errors were discovered in the previous work and up to the present the effect of the correction seemed to have been in every case to increase the amount of energy diffracted around the globe.

Prof. C. H. LEES mentioned that in the formula $(\mu - 1)/q = A$, the value assumed for μ was for sodium light, whereas the value required was for $\lambda = 500$ metres.

Prof. MARCHANT mentioned that in some work recently published it was shown that the difference between the carrying power of signals by night and by day was very much greater in summer than in winter. He asked within what limits of accuracy Gladstone and Dale's law held.

Prof. FLEMING, in reply, said that Dr. Chree's remarks were of importance, and the questions raised required to be cleared up. In reply to Dr. Lees, he said that for all gases the square root of the dielectric constant for steady voltage practically agreed with the refractive index for sodium light. It would be useful to have the known facts as to the nature of the atmosphere collected for the use of wireless investigators.

XXXIV. *Atmospheric Electricity Observations made at Kew Observatory, by GORDON DOBSON, B.A.*

COMMUNICATED BY DR. CHREE. RECEIVED JUNE 5, 1914.

DURING the past few years the observations of atmospheric electricity at Kew Observatory have included measurements with Prof. Ebert's apparatus for the number of ions per cubic centimetre and for the electric conductivity of the air, and also with Mr. C. T. R. Wilson's apparatus for the electric conductivity of the air and the vertical air-earth current. As some doubt had been raised as to the exact nature of the results given by this latter instrument, and also about the method of using it which had been employed at Kew, some observations were made to try to settle these points.

In the instrument designed by Prof. Ebert air is drawn through a vertical earthed tube, in the axis of which is an insulated rod connected to an electrometer and charged to about 200 volts. As the air passes through the tube ions of the opposite sign to the charge on the rod will be caught by it, and the loss of charge on this rod measures the charge received from the ions in the volume of air drawn through. Allowance is made for leakage over the insulators supporting the charged rod.

The mobility of the ions is measured by placing a short rod, charged to a low potential, in the air current before it reaches the main rod. Some ions will be caught by this rod, so that the number caught by the main rod will be smaller than before. Assuming the total number of ions to remain constant, the difference in the number caught by the main rod in the two cases will be equal to the number caught by the auxiliary rod. Since the proportion of ions which are caught on the auxiliary rod depends on its potential and the mobility of the ions the mobility of the ions can thus be obtained.

With this apparatus we can, therefore, measure both the charge per unit volume of air and also the mobility of the ions. The conductivity of the air depends on these two values alone. If λ_+ be the conductivity due to the positive ions, E_+ the charge on them per unit volume, and u_+ their mobility, we have

$$\lambda_+ = E_+ u_+,$$

and, similarly, for the negative ions

$$\lambda_- = E_- u_-.$$

The total conductivity of the air is given by

$$\lambda = \lambda_+ + \lambda_-.$$

Unfortunately, the average conductivity of the air at Kew is very small, so that, with the electrometer which is usually fitted to the Ebert apparatus, it frequently happens that the conductivity is too small to be measured by this method. Also, since the value for the mobility depends on the difference between the results of two observations, and, since the number of ions is always changing, it not infrequently happens that the conductivity comes out negative.

The conductivities obtained by the Ebert apparatus at Kew are, therefore, only very approximate values, but the mean of a large number of observations may be fairly reliable. Table I. shows how often zero or negative values are given by this instrument at Kew :—

TABLE I.

—	Year.	Winter.	Summer.
Total number of observations.....	107	41	66
Number of cases in which zero or negative values were obtained for both u_+ and u_-	14	12	2
Ditto for u_+	25	17	8
Ditto for u_-	43	25	17
Ditto for either u_+ or u_-	54	31	23

In Mr. Wilson's apparatus* the electric current is measured which enters a small test plate, freely exposed to the earth's field and kept at zero potential. This test plate is surrounded by a guard-ring and may be kept at any desired potential by a charged sliding condenser or "compensator." The potential gradient immediately above the test plate can be obtained by measuring the charge induced on it when freely exposed. If the instrument could be used so that the test plate were level with, and practically continuous with the surface of flat open ground, then both the potential gradient above it and the current entering it would be the same as for any other part of the surrounding ground.

In practice it is not convenient to use the instrument under these conditions, and it is, therefore, placed on a stand. In the observations taken at Kew the test plate has always been approximately 135 cm. above the ground. Under these con-

* Cambridge Phil. Soc. "Proc.," Vol. XIII., pp. 184 and 364.

ditions the potential gradient above the test plate is very greatly increased and, if the conductivity of the air is everywhere the same the current entering the test plate will be increased in the same proportion. If the test plate were always used at the same height above the ground it would be possible to obtain a factor showing how much the potential gradient above the test plate was greater than that above the surface of the ground. If, therefore, we assume that the conductivity is everywhere the same we can apply this factor to the measured current entering unit area of the test plate and obtain the current entering the ground.

For various reasons this method has not been used at Kew; and advantage has been taken of the fact that the potential gradient immediately above the test plate can easily be measured, and the conductivity has been calculated from the current entering the test plate and the potential gradient immediately above it. This method gives a result which can be directly compared with the conductivity as given by the Ebert apparatus. If the air-earth current be required it can be obtained from this conductivity and the potential gradient given by the recording electrograph.

It has been assumed by Dr. Lutz* that, since in this apparatus the current measured is due to the positive ions, it will measure only the part of the total conductivity denoted above by λ_+ . Assuming λ_+ and λ_- to be equal—a result which seems to be approximately true at many stations, though, as will be seen from tables 10 and 11, it is not true at Kew—it is supposed that Wilson's apparatus measures about half the conductivity and half the current. It has further been suggested that when this instrument is used on a stand, since the potential gradient above the test plate is thereby greatly increased, conditions approaching saturation may be obtained, and the value given for the conductivity be, therefore, too small.

It seems probable that if the Wilson apparatus could be used with its test plate practically continuous with the general surface of the ground all these doubtful points would disappear. The total current passing through the air must be equal to that entering the ground, and the potential gradient could not be changed from its normal value, so that no saturation effects could occur. It was not found practicable to use

* *Luftelektrische Messungen*, Munich, 1905-10.

the instrument in a pit with its test plate level with the ground ; so observations were made with an entirely different test plate. This was made of wood covered with tinfoil, and insulated on sulphur. The test plate was 29 cm. square, and was surrounded by a guard-ring 15 cm. broad. The whole was in the form of a very shallow box, 4 cm. deep. Connection could be made with the test plate through one side of the box. The box was sunk into the ground, so that the test plate and guard-ring were level with the surrounding ground. The test plate was connected to the electrometer of the ordinary Wilson apparatus, which was placed on a low stand some little distance away. The connecting wire was carried on sulphur insulators inside an earthed metal tube, as it was necessary to shield it from the earth's field. A metal cover could be placed over the test plate, and the apparatus was used in the same way as that

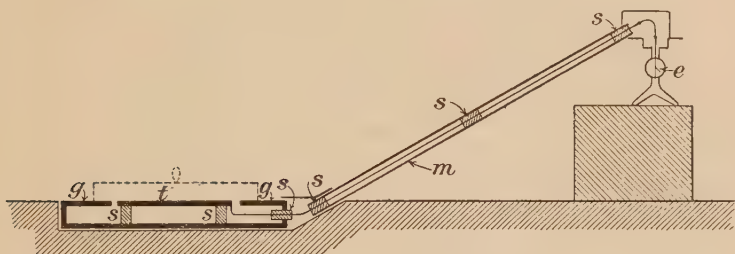


FIG. 1.

having the usual small test plate. The general arrangement of the apparatus is shown in Fig. 1.

During the spring of 1912 some observations were made with this apparatus, alternate measurements being made using this large test plate, and using the instrument with its usual small test plate, and placed on the stand which is ordinarily used. Owing to the constant variation of the electrical conditions, it was found that a very long series of observations would be necessary to get satisfactory results by this method. In the summer a second Wilson apparatus was obtained, and the experiments were begun again in September, simultaneous observations being made with the two instruments, one connected to the large test plate at ground level and the other used in the normal way on the stand. When the large test plate is used, the deflections of the electrometer are smaller than when the instrument is used in the ordinary way, so that

the readings are less accurate. Every set of observations consists of six or more simultaneous observations with the two instruments. Both instruments in turn were used in connection with the large test plate.

At first, observations were made with the two instruments on two stands a short distance apart, both being used in the ordinary way with their usual test plates. When the insulation was satisfactory on each instrument the results were in good agreement; the means of four sets, each of six simultaneous observations taken on four different days being as follows :—

TABLE II.

Conductivity.	
Instrument "A."	Instrument "B."
0.51×10^{-25} E.M.U.	0.50×10^{-25} E.M.U.
0.44 "	0.52 "
0.75 "	0.76 "
0.47 "	0.46 "
0.54 " (Mean)	0.56 " (Mean)

The means of each series of observations probably agree as well as could be expected, but the individual observations are not always in such good agreement. This may be partly due to the fact that the observations are not strictly simultaneous, one instrument being read first and the other as soon afterwards as possible. As a typical set of observations, the first of the above table is given below :—

TABLE III.

Conductivity.		
—	Instrument "A."	Instrument "B."
Obs. No. 1.....	0.55×10^{-25} E.M.U.	0.71×10^{-25} E.M.U.
" 2.....	0.66 "	0.55 "
" 3.....	0.46 "	0.45 "
" 4.....	0.52 "	0.46 "
" 5.....	0.45 "	0.46 "
" 6.....	0.45 "	0.40 "
Means	0.51 "	0.50 "

The results of the simultaneous observations with two Wilson instruments, one being connected with the large test plate at

ground level, and the other used on the stand in the ordinary way, are given in Table IV. :—

TABLE IV.

Number of observations.	Instrument which was used with large test plate.	Conductivity.		Ratio of conductivity from Large test plate Small test plate.
		Large test plate at ground level.	Usual test plate instrument on stand.	
7	A	0.28×10^{-25} E.M.U	0.23×10^{-25} E.M.U	1.00
5	B	0.25 "	0.23 "	1.09
6	B	0.37 "	0.31 "	1.19
6	A	0.39 "	0.25 "	1.56
6	B	0.55 "	0.47 "	1.17
5	B	0.48 "	0.38 "	1.26
Means		0.39 "	0.32 "	1.22

It appears that the values for the conductivity given by the apparatus with the large test plate at ground level are some 20 per cent. larger than those given by the Wilson apparatus when used in the normal way on the stand. Tables II. and III. show the agreement between two similar instruments, and it could not be expected that the accuracy of the readings when using the large test plate would be quite so great as when using the instrument in the ordinary way, since the observations are more difficult to make. There is, however, no reason to suppose that there is any systematic error in these observations ; so that the means should be trustworthy. The following table shows how a typical set of observations (the fifth in the above table) agree among themselves :—

TABLE V.

Conductivity.		
Observation No.	Large test plate.	Usual test plate.
1	0.62×10^{-25} E.M.U	0.41×10^{-25} E.M.U.
2	0.60 "	0.43 "
3	0.50 "	0.54 "
4	0.62 "	0.56 "
5	0.51 "	0.42 "
6	0.43 "	0.50 "
Means	0.55 "	0.47 "

Table VI. gives the values obtained, in the spring of 1912, for the conductivity, using the large, and usual, test plates.

At this time, only one Wilson instrument was available, and observations were, therefore, taken alternately, first with the instrument on its usual stand with its ordinary test plate, and then with the electrometer connected to the large test plate at ground level. These figures are, naturally, not so accurate as the results obtained by taking simultaneous observations with two instruments. They show, however, decidedly higher values when the large test plate is used :—

TABLE VI.

Number of pairs of observations.	Conductivity.		Ratio of conductivity Large test plate Small test plate.
	Large test plate at ground level.	Usual test plate instrument on stand.	
3	0.20×10^{-25} E.M.U.	0.11×10^{-25} E.M.U.	1.82
3	0.15 "	0.11 "	1.36
4	0.23 "	0.17 "	1.35
3	0.16 "	0.22 "	0.73
4	0.31 "	0.20 "	1.55
4	0.40 "	0.18 "	2.22
4	0.15 "	0.18 "	0.84
4	0.36 "	0.25 "	1.44
5	0.32 "	0.31 "	1.03
Means	0.25 "	0.19 "	1.32

It would have been more satisfactory if more observations could have been made, but, unfortunately, the weather during the summer of 1912 was not good for these experiments. Each observation lasts 10 minutes, and the time occupied in getting a set of six observations, including setting up and taking down the apparatus, is over two hours. Although the accuracy of the observations is not so great as could be wished for, yet it seems fairly certain that, when the conductivity is measured by using the large test plate at ground level, higher values are obtained than when the instrument is used in the ordinary way on the stand. The difference seems to be about 20 per cent.

A few sets of observations were made, in which both instruments were used in the normal way with their ordinary test plates, but one instrument stood on the ground while the other stood on the usual stand. In each set of observations half the observations were made with one instrument on the stand

and half with the other on the stand. The following values of conductivity were obtained :—

TABLE VII.

Number of observations.	Conductivity.		Ratio
	Instrument on ground.	Instrument on stand.	$\frac{\text{Ground}}{\text{Stand.}}$
12	0.365×10^{-25} E.M.U.	0.330×10^{-25} E.M.U.	1.11
12	0.255 "	0.225 "	1.13
9	0.275 "	0.240 "	1.15
Means ...	0.295 "	0.265 "	1.12

When the instrument stands on the ground the test plate is about 35 cm. above the ground. The observations show very good agreement among themselves, and also show the increased value of the conductivity when measured near the ground.

In the cases when the large test plate was used at ground level, or when the ordinary instrument stands on the ground, the potential gradient is much less, and the equipotential surfaces above the test plate are much less curved than when the instrument is used on the stand. It, therefore, seemed of interest to put a very large guard-ring round the ordinary test plate, using the instrument on the ordinary stand. In this way the potential gradient immediately above the test plate is reduced, and the equipotential surfaces must be nearly flat immediately above it. For this purpose a wooden guard-ring, 30 in. square and covered with tinfoil, was placed on the ordinary guard ring. Simultaneous observations were made with two instruments, both on stands of the usual height, one instrument having the large guard-ring and one having only its ordinary small guard-ring. The large guard-ring was half the time on one instrument and half on the other. The values for the conductivity obtained in the two ways are given in Table VIII. :—

TABLE VIII.

Number of observations.	Conductivity.		Ratio
	Large guard-ring.	Small-guard ring.	$\frac{\text{Large}}{\text{Small}}$
10	0.390×10^{-25} E.M.U.	0.295×10^{-25} E.M.U.	1.31
7	0.280 "	0.245 "	1.13
12	0.275 "	0.250 "	1.10
Means	0.315 "	0.265 "	1.19

The observations are not in very good agreement, but they show that the value found for the conductivity is certainly greater when a large guard-ring is used than when the ordinary small guard-ring is used.

If there were any real difference between the conductivity of the air near the ground and that a few metres above it we should expect that there would be a change in the number of ions, since it is hardly likely that the mobilities would be different in the two places. A difference in the number of ions in the air near the ground and in that a few metres higher might be caused either by the migration of ions under the electric potential gradient or by highly ionised air coming out of the ground. It seems probable that, with even a moderate wind, the air would be stirred up sufficiently to prevent the formation of any layer of different ionisation near the ground. At Dr. Chree's suggestion, several observations of the number of ions were made with the Ebert apparatus on calm days, taking air from the surface of the ground and from about 2 metres above it. The results are given in Table IX., the last column of which table gives the direction and estimated Beaufort force of the wind at the time when each set of observations were taken :—

TABLE IX.

Positive charge per c.c. E.M.U. $\times 10^{20}$.		Negative charge per c.c. E.M.U. $\times 10^{20}$.		Wind
Ground.	2 metres.	Ground.	2 metres.	
150	180	180	140	S. 0 to 1
380	360	W. 1 to 2
...	...	340	270	W. 1
...	...	390	420	Calm
...	...	320	330	S. 1
400	400	300	330	S. 1
440	330	330	360	S.W. 1
140	180	180	180	N.W. 1
302	290	291	290	Means.

Although the observations were made on days with very little wind no appreciable difference in the ionisation of the air from near the ground and that from 2 metres above it is shown.

It is interesting to compare the values of the conductivity given by the Wilson apparatus with those from observations made at the same time with the Ebert apparatus. Unfortunately, as stated above, owing to the low average conduc-

tivity at Kew, the individual values given by the Ebert apparatus are not satisfactory. But, by taking a large number of observations for the conductivity by the Ebert and by the Wilson instruments we should be able to get a fairly satisfactory comparison from the means. Table X. gives values obtained from the Ebert apparatus for λ_+ , λ_- and $(\lambda_+ + \lambda_-)$, and also the conductivity given by the Wilson apparatus at the same time. All days were excluded from this table on which the value of either λ_+ or λ_- was zero or negative. The Wilson instrument was used on its stand, with its ordinary guard-ring, in these observations. The values obtained from it, therefore, require a correction of 20 per cent.; this correction has been applied only to the final mean:—

TABLE X.

Conductivity in E.M.U. $\times 10^{25}$									
Date.	Ebert.			Wilson.	Date.	Ebert.			Wilson.
	λ_+	λ_-	$\lambda_+ + \lambda_-$			λ_+	λ_-	$\lambda_+ + \lambda_-$	
1912									
Feb. 27	0.26	0.21	0.47	0.36	June 19	0.67	0.28	0.95	0.75
Apr. 19	0.52	0.28	0.80	0.72	21	0.70	0.50	1.20	0.66
22	0.22	0.27	0.49	0.37	27	0.44	0.20	0.64	0.58
25	0.27	0.14	0.41	0.39	July 8	0.75	0.47	1.22	0.51
May 2	0.22	0.35	0.57	0.47	10	0.96	0.43	1.39	0.81
6	0.41	0.48	0.89	0.70	11	0.27	0.67	0.94	0.77
15	0.51	0.52	1.03	0.73	12	0.37	0.30	0.67	0.98
22	0.52	0.32	0.84	0.55	18	0.68	0.34	1.02	0.45
24	0.42	0.14	0.56	0.44	23	0.23	0.54	0.77	0.65
June 11	0.62	0.20	0.82	0.34	26	0.78	0.22	1.00	1.02
17	0.45	0.55	1.00	0.51	30	0.80	0.41	1.21	0.66
18	0.54	0.44	0.98	0.44	Aug. 1	0.20	0.68	0.88	0.56
					Sept. 30	0.28	0.41	0.69	0.40
					Means	0.484	0.374	0.858	0.593
					Wilson value + 20% =			0.712	

The mean from this table gives a value for λ_+ from the Ebert apparatus, which is about 35 per cent. below the conductivity (corrected by 20 per cent.) given by the Wilson apparatus; while the value for $(\lambda_+ + \lambda_-)$ is somewhat above the corrected Wilson value. Owing to the inaccuracy of the reading with the Ebert apparatus we may suppose that the values given by it are as often too high as too low. Now, many of the readings which are too low will be zero or negative and are not included in this table, while all the readings which are too high are included. This will make the means from the Ebert apparatus

somewhat too high. It is difficult to know how much the values from the Ebert apparatus in Table X. are increased by selecting only those days with positive values for λ_+ and λ_- . If we divide the sum of the measurable conductivities by the total number of days on which observations were made we shall get a value which was too low, since it is highly improbable that the conductivity was ever really zero. If, however, the days with zero or negative values are few it would seem that the mean thus obtained should be only slightly below the true value. In Table XI. the observations on 56 days are given. These are all the observations made during the seven months of the year when the conductivity is greatest. On

TABLE XI.

Conductivity in E.M.U. $\times 10^{-25}$									
Date.	Ebert.			Wilson.	Date.	Ebert.			Wilson.
	λ_+	λ_-	$\lambda_+ + \lambda_-$			λ_+	λ_-	$\lambda_+ + \lambda_-$	
1912.									
Mar. 14	0.06	—	0.06	0.31	May 28	—	0.28	0.28	0.52
20	0.44	—	0.44	0.24	29	0.49	—	0.49	0.63
22	0.30	—	0.30	0.30	31	0.13	0.07	0.20	0.51
27	0.26	0.21	0.47	0.24	June 5	—	0.07	0.07	0.50
28	—	0.35	0.35	0.45	10	0.62	0.20	0.82	0.34
29	0.21	—	0.21	0.28	13	0.35	—	0.35	0.21
Apr. 2	—	—	—	0.21	14	0.20	—	0.20	0.38
3	—	—	—	0.29	17	0.45	0.55	1.00	0.51
11	0.03	—	0.03	0.15	18	0.54	0.44	0.98	0.44
12	0.24	0.04	0.28	0.26	19	0.67	0.28	0.95	0.75
15	—	—	—	0.24	20	0.07	0.48	0.55	0.47
16	0.30	—	0.30	0.26	21	0.70	0.50	1.20	0.66
18	0.38	—	0.38	0.47	27	0.44	0.20	0.64	0.58
19	0.52	0.28	0.80	0.72	July 6	—	0.46	0.46	0.24
22	0.22	0.27	0.49	0.37	8	0.75	0.48	1.23	0.51
23	0.23	—	0.23	0.44	10	0.96	0.43	1.39	0.81
24	0.06	—	0.06	0.34	11	0.27	0.64	0.91	0.77
25	0.34	0.14	0.48	0.39	12	0.37	0.30	0.67	0.98
30	—	0.14	0.14	0.56	18	0.68	0.34	1.02	0.45
May 1	0.41	—	0.41	0.48	23	0.23	0.53	0.76	0.65
2	0.21	0.35	0.56	0.47	25	0.56	—	0.56	0.79
3	0.43	—	0.43	0.42	26	0.78	0.25	1.03	1.03
6	0.41	0.48	0.89	0.70	30	0.80	0.41	1.21	0.66
15	0.51	0.52	1.03	0.73	Aug. 1	0.20	0.07	0.27	0.56
16	0.07	0.14	0.21	0.55	Sept. 18	0.86	—	0.86	0.36
22	0.52	0.32	0.84	0.55	19	0.28	—	0.28	0.13
23	0.14	—	0.14	0.58	20	0.39	—	0.39	0.29
24	0.42	0.14	0.56	0.44	30	0.28	0.41	0.69	0.40
					Means	0.336	0.192	0.538	0.475
						Wilson value + 20% =			0.570

most of the days both λ_+ and λ_- had positive values. The mean gives a value for λ_+ which is very much below the corrected value from the Wilson apparatus; but the sum of λ_+ and λ_- is only very slightly below the Wilson value. If we take only the observations made in the summer months there are very few observations when either λ_+ or λ_- had either zero or negative values. The mean for $(\lambda_+ + \lambda_-)$ for the three summer months is 0.873×10^{-25} E.M.U.; while that for λ_+ is 0.498×10^{-25} E.M.U.; and the corresponding value from the Wilson apparatus (corrected by 20 per cent.) is 0.752×10^{-25} E.M.U. A set of days has also been selected on which the conductivity as measured by the Wilson apparatus was high, those days with values above 0.40×10^{-25} E.M.U. being used. These days gave a mean for $(\lambda_+ + \lambda_-)$ equal to 0.659×10^{-25} E.M.U., that for λ_+ , 0.389×10^{-25} E.M.U., and the corrected value from the Wilson apparatus 0.712×10^{-25} E.M.U.

There seems to be little doubt, therefore, that the corrected conductivity obtained from the Wilson apparatus corresponds with $(\lambda_+ + \lambda_-)$ as measured by the Ebert apparatus. This is, indeed, what we should have expected from the results with the large test plate at ground level.

In using the Ebert apparatus, all the small fast-moving ions will be caught on the main rod, but some of the large, slow-moving ions will also be caught. If we assume the figures given by Langevin—namely, that the number of the large ions is about 50 times that of the small ions, while their mobility is about 1,000 times less—then, as the Ebert apparatus is used at Kew, about 20 per cent. of the value found for the number of ions per unit volume of air, is due to large ions. The value of the mobility, however, is decreased nearly 20 per cent. below the true mobility of the small ions by the presence of the large ions; the effect on the measured conductivity being that it is about 1.7 per cent. greater than if no large ions were present. Under the same conditions about 1.7 per cent. of the current entering the test plate of the Wilson apparatus will be due to the large ions, so that the conductivity, as measured by it, is also 1.7 per cent. greater than if no large ions were present.

Mr. Wilson has described* experiments to test whether placing grass on the test plate of his apparatus had any effect on the results. In these experiments a small piece of turf was placed on the ordinary small test plate of the apparatus,

* "Proc." Roy. Soc., Vol LXXX., p. 537.

which stood on a low stand. The observations showed that the grass had no appreciable effect. No attempt, however, was made to measure the current which entered a large test plate at ground level. The experiments described above do not, therefore, contradict the results of these observations, but seem to point to the fact that the assumption is not quite correct that "if the ratio of the current to the charge on the exposed surface were found to be the same in the two cases" (with and without grass on the test plate) "it might with some confidence be assumed that the same ratio would hold for the current to the charge per unit area of the ground."

Some observations were begun to measure the conductivity of the air, using the large test plate, both it and its guard-ring being covered with turf, and sunk into the ground, so that the turf was practically continuous with that surrounding it. The observations in this way were considerably more difficult to make, and it was thought better to make observations at first with the large test plate not covered with turf. These observations took all the available time during the fine weather of the summer of 1912, so that it has not been possible to make further observations with the large test plate covered with turf. The results of Mr. Wilson's experiments with turf on the ordinary test plate of the instrument would lead us to expect, however, that there would be no difference if the large test plate were covered with turf.

ABSTRACT.

The Paper gives an account of experiments made to determine the accuracy of the results obtained with the apparatus designed by Mr. C. T. R. Wilson for measuring the electric conductivity of the air, and the electric current passing from the air to the earth. Observations were made (1) using the standard instrument on a stand according to the usual practice and (2) using an experimental apparatus level with the ground, which was assumed to give correct results. It was found that it was necessary to apply a small correction to the results obtained with the standard apparatus when used in the ordinary way.

A comparison was also made of the electric conductivity of the air as measured by Mr. Wilson's apparatus and that designed by Prof. Ebert. The results given by this latter apparatus appear to be too inaccurate to allow any satisfactory conclusions to be drawn.

DISCUSSION.

Prof. C. H. LEES said it was important to settle which was the better of the two methods. He thought the large Langevin ions were at the root of the trouble.

Dr. CHREE said the large ions considerably affected the charge per unit volume, but he did not think they would affect the value of the earth-air current.

XXXV. *Thermal and Electrical Conductivities of Some of the Rarer Metals and Alloys.* By THOMAS BARRATT, A.R.C.S., B.Sc.

RECEIVED JUNE 3, 1914.

I. *Introduction.*

THE following investigation was undertaken, not with the idea necessarily of giving more accurate determinations of the quantities involved than those given in the most recent researches,* but in the first place because the method can be employed for many of the rarer metals and alloys, and, secondly, because the mathematics, and especially the formulæ used, are exceedingly simple.

In most of the investigations on thermal conductivity one end of a rod or wire enclosed in a water jacket,† or open to the air,‡ is heated, and measurements taken of the amount of heat supplied to one end of the specimen, and of two or more temperatures at various points along its length. For example, Lees measured the heat given (electrically) and two temperatures; while Jäger and Diesselhorst measured three temperatures at fixed points along the rod. In each of the two researches just mentioned one end of the rod was securely fastened to a copper block forming part of the enclosure. The presence of the thermometers necessary for measuring the temperatures at various points along the rod involved elaborate precautions and corrections, which in the present research are avoided, the specimen being completely bare from end to end. Another type of arrangement of apparatus is that introduced by F. A. Schulze,§ and employed by Grüneisen|| and others. In this method, which may be termed the "variable temperature" method, the rod is allowed to assume a constant temperature, usually that of the enclosure, and then one end is suddenly cooled—*e.g.*, by a stream of cold

* C. H. Lees, Bakerian Lecture, Roy. Soc., 1908; W. Jäger and H. Diesselhorst, "Abh. d. Phys., Tech. Reich.," 3, 269, 1900; L. Lorenz, "Ann. d. Physik," XIII., p. 422, 1881.

† Lees, Jäger and Diesselhorst, *loc. cit.*; Wiedemann and Franz, "Ann. d. Phys.," LXXXIX., p. 497, 1853.

‡ Forbes, Edin. "Trans.," XXIV., 73, 1867. R. W. Stewart, "Proc.," Roy. Soc. Lond., LIII., 151, 1893; Biot, "Traite de Phys.," 1816.

§ F. A. Schulze, "Wied. Ann.," LXVI., 2, p. 207, 1898.

|| E. Grüneisen, "Ann. d. Physik," III., 1, pp. 43-74, Sept., 1900.

water. The temperature is then observed at regular intervals, a few centimetres from this end, by means of a thermojunction. Unfortunately, the results obtained by this method do not, as a rule, agree at all well with those given by the "stationary temperature" method. For example, Schulze obtained as a mean result for copper the value 0.6108, which is 30 per cent. lower than the present accepted value. It has been pointed out by Schaufelberger* that the discordant results given by Schulze's method are, partly at any rate, due to the unjustifiable assumption that the end of the heated bar acquires the temperature of the water by which it is cooled. This assumption will easily lead to an error of 10 per cent.

Still another method of procedure has recently been employed by Angell† at the suggestion of C. E. Mendenhall. A rod of the metal was electrically heated, and the temperature measured at the centre and at the circumference of the rod by means of platinum and platinum-rhodium couples. The method was used principally for the determination of thermal conductivities at high temperatures. Gray‡ used specimens (as in the present research) in the form of thin wires, but the heat was supplied by steam, and the amount of heat flowing along the wire was measured by the rise of temperature of a copper sphere of known thermal capacity suspended from the lower end of the wire. His method was a combination of the "stationary" and "variable" types. His results were given as at a temperature 10°C. to 97°C.

Reference is made to some of the results obtained by the above-mentioned experimenters in the tables given towards the end of the present Paper.

An exhaustive account of researches on thermal conductivity up to 1906 is given in Winkelmann's "Handbuch der Physik," volume "Wärme"; also in a French edition of Chwolson's "Physics," which has been recently published. At present there appears to be no English publication giving anything like an adequate account of the subject.

II. Apparatus and Measurements.

The method employed in the present research is a new one, of the "stationary temperature" type. Measurements are

* W. Schaufelberger, "Ann. d. Physik.," VII., 3, pp. 589-630, 1902.

† M. F. Angell, "Phys. Rev.," XXXIII., pp. 421-432, Nov., 1911.

‡ Gray, "Proc." Roy. Soc. London, LVI., 205, 1894.

made of (a) the heat supplied electrically to one end of the metal; (b) the temperatures of this end and of the enclosure. The method is particularly applicable to metals, in the form of wires, which cannot easily be obtained in bulk, owing to scarcity or cost. Such metals as tungsten, molybdenum, iridium, rhodium and tantalum have, therefore, been tested. Platinum-iridium and platinum-rhodium have also been included, as a knowledge of their thermal conductivity is likely to be useful to experimenters employing them as thermo-electrical thermometers. The elasticity of the method is illustrated by its extension to a non-metal—viz., graphite—and experiments are already in progress with a modified form of apparatus, on the measurement of the thermal conductivity of other more or less badly-conducting substances, including, if possible, liquids. Experiments have been carried out at laboratory temperatures and at 100°C., and the method could easily be extended to temperatures above or below those mentioned.

It will be shown in the next section that the thermal conductivity k is given by the equation,

$$k = \frac{H^2}{pqhV^2} \coth^2 al,$$

where H is the amount of heat flowing into the wire at the hot end; p is the perimeter, q the cross-sectional area of the wire; V the excess of temperature of the hot end over that of the enclosure; h the amount of heat lost per second from 1 sq. cm. of the wire when its temperature is 1°C. above that of its surroundings; and $\alpha = \sqrt{\frac{hp}{qk}}$. When l is great the equation reduces to the very simple form:—

$$k = \frac{H^2}{pqhV^2}$$

The wires employed were nearly all of diameter about 1 mm., and of lengths ranging from 30 cm. upwards, so that either the simple form of equation could be employed or the factor $\coth^2 al$ was an exceedingly small correction.

IIa. Measurement of H and V .

Fig. 1 illustrates the method employed in the determination of H , the amount of heat flowing into the specimen at its

hot end, and of V, the excess of temperature of this end above that of the enclosure. The specimen AB was "coppered" at the end A by electrolytic deposition, and this end carefully filed until it fitted tightly into a slightly conical hole, 1.3 mm. diameter at its widest part, bored in a small copper cylinder of length 9 mm. and diameter 6 mm. at the end A. This cylinder was enclosed by a hollow cylinder, AC, made of thin copper, and of length 4 cm. Round the latter was wound just 3 metres of single silk-covered pure platinum wire, of gauge 36, whose resistance at 17°C. was 24.25 ohms. Through this platinum coil a measured current, C, was passed. The platinum wire coil was covered with fine silk paper, thinly coated with shellac varnish.

The E.M.F. (E) at the ends of this wire was carefully measured by means of a Rayleigh potentiometer, every coil of which has been calibrated in terms of a standard resistance. The current C was also measured at the same time and in the

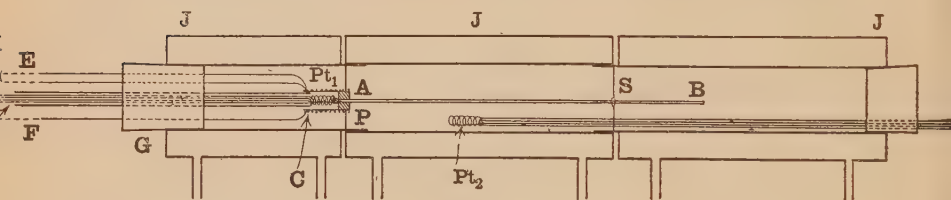


FIG. 1.

same way by obtaining the E.M.F. at the ends of a standard resistance coil O (Fig. 3), through which the current also passed. Thin silk-covered copper wires E and F, of gauge 30, joined to the ends of the platinum coil round AC were brought through the cork G (Fig. 1), as was also a third wire, H, of platinoid No. 30, to serve as potential lead. Inside the cylinder AC was a platinum thermometer, Pt_1 , which, in connection with a Callendar-Griffiths bridge, G (Fig. 3), gave the temperature of the hot end A of the wire. Another similar thermometer, Pt_2 , indicated the temperature of the enclosure. This consisted of a brass water-jacket, JJJ, in three parts, of inner diameter 9 cm., and total length 60 cm., enclosed also in cotton wool. The corresponding readings of the two platinum thermometers were frequently verified. Each was provided with compensating leads exactly similar to those in connection with the corresponding thermometer, and lying

side by side with them. The four leads from Pt_1 of thin double silk-covered copper wire, gauge 30, were carried through a glass tube CD of length 25 cm., which also served to keep the end A of the wire in the middle of the water-jacket. The specimen could be taken out of the copper cylinder, or replaced,

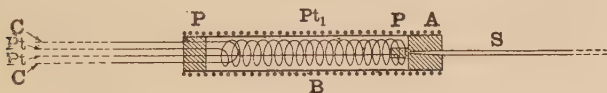


FIG. 2.

in a few seconds. It was held in position in the centre of the water-jacket by a very thin silk thread at S.

Fig. 2 illustrates on a larger scale the relative positions of the specimen wire S, copper cylinders A and B, platinum

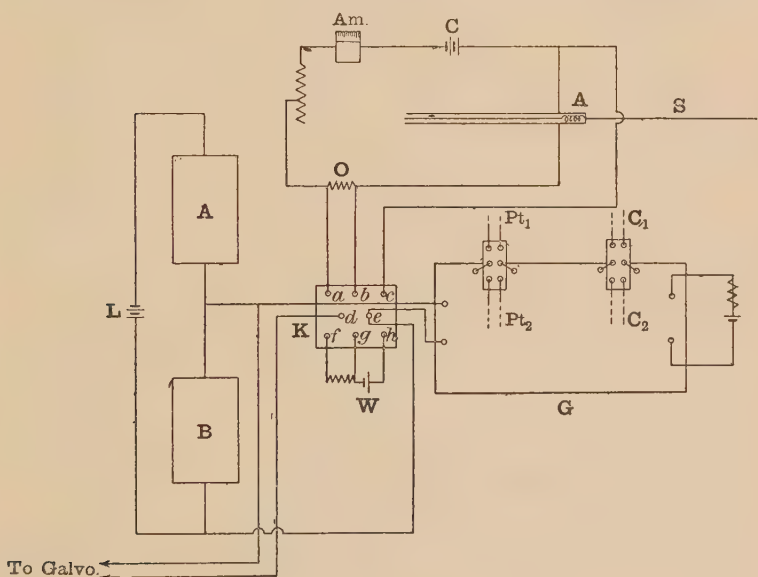


FIG. 3.

coil PP, platinum thermometer Pt_1 and compensating leads CC.

The electrical connections are shown in Fig. 3, G being the Callendar-Griffiths bridge, AB the Rayleigh potentiometer, O the standard 1 ohm coil, and K a key to enable one to com-

pare the current C and E.M.F. E with a Weston cell, W . The arrangement allows one to use the same galvanometer for all purposes. E is obtained in terms of W by connecting bd , ce , and then fd (or gd), eh . C is obtained in terms of W by joining ad , be , and then fd (or gd), eh .

The excess of temperature V of the hot end of the wire over that of the enclosure was given by the readings of Pt_1 and Pt_2 . The latter thermometer also gave the actual temperature of the enclosure. For the measurement of H , the heat flowing in at the hot end of the wire, the latter was adjusted with this end in the copper block, and the amount of heat CE/J (where CE is in "Joules" and $J=4.185$) that was required to raise the temperature of the copper cylinder V° above that of the jacket was measured, ample time being given after the current was switched on to allow temperature conditions to become steady. The specimen was then removed. It was found that a considerably smaller amount of heat, $C'E'/J$, was now required to maintain the same temperature difference, V° , between the end A of the wire and the enclosure. The value of the expression $(CE-C'E')/J$ gives the amount of heat H which is given to the wire, and is lost by convection and radiation from the sides and the other end of the wire. It is worthy of notice that the heat lost from the copper cup and the coil surrounding it is the same whether the specimen is in place or not (apart, that is, from the heat that is given to the wire), as the temperature difference between the coil and its surroundings is maintained the same in both cases. Thus, all the heat given by the expression $(CE-C'E')/J$ flows into the specimen, and is transferred thence to the water jacket. The method, therefore, has the advantage of being a null one. In addition to this, it may be mentioned that no corrections are necessary (as in other methods) for loss of heat from thermometer leads, which would also involve a distortion of the lines of flow of heat. The wire is entirely uncovered, all the necessary temperatures being measured at points quite apart from the metal itself. Finally, the experiment takes a very short time once the conditions have become steady, and, owing to the very simple formula, the calculations involve very little time or labour.

III. Theory of the Experiment.

When a thin rod of cross-section q and perimeter p is heated at one end, and loses heat from its surface and the other end, the

excess of temperature v at a distance, x , from the hot end over that of its surroundings is given in the steady state by

$$\frac{d^2v}{dx^2} = \frac{hp}{qk} \cdot v = (\text{say}) \alpha^2 v, \quad \dots \dots \dots (1)$$

where k is the thermal conductivity, h the heat lost per second per unit surface area per 1°C . difference of temperature between the rod at that point and its surroundings, and $\alpha = \sqrt{\frac{hp}{qk}}$.

The solution of this equation is

$$v = A \cosh \alpha x + B \sinh \alpha x. \quad \dots \dots \dots (2)$$

At the point $x=0$ let $v=V$; then $A=V$.

$$\text{Hence} \quad v = V \cosh \alpha x + B \sinh \alpha x, \quad \dots \dots \dots (3)$$

$$\text{and} \quad \frac{dv}{dx} = \alpha V \sinh \alpha x + \alpha B \cosh \alpha x. \quad \dots \dots \dots (3A)$$

If the rod is of length l , we have at the point $x=l$,

$$kq \frac{dv}{dx} + hqv = 0. \quad \dots \dots \dots (3B)$$

$$\therefore k\alpha(V \sinh \alpha l + B \cosh \alpha l) + h(V \cosh \alpha l + B \sinh \alpha l) = 0$$

$$\text{or} \quad B = -V \frac{k\alpha \sinh \alpha l + h \cosh \alpha l}{k\alpha \cosh \alpha l + h \sinh \alpha l}.$$

Hence, from (3),

$$v = V \left\{ \cosh \alpha x - \frac{k\alpha \sinh \alpha l + h \cosh \alpha l}{k\alpha \cosh \alpha l + h \sinh \alpha l} \cdot \sinh \alpha x \right\}. \quad \dots \dots (4)$$

This equation holds whether l is great or small. If l is great the equation reduces to

$$v = V e^{-\alpha x}. \quad \dots \dots \dots (5)$$

If not, we have from equation (4)

$$v = V \frac{k\alpha \cosh \alpha(l-x) + h \sinh \alpha(l-x)}{k\alpha \cosh \alpha l + h \sinh \alpha l}. \quad \dots \dots (4A)$$

At the point $x=0$, $H = kq \left(-\frac{dv}{dx} \right)$. Hence, from (4A),

$$\begin{aligned} H &= kqV\alpha \frac{k\alpha \sinh \alpha l + h \cosh \alpha l}{k\alpha \cosh \alpha l + h \sinh \alpha l}, \\ &= kqV\alpha \tanh \alpha l, \text{ if } h/k\alpha \text{ is small.}^* \end{aligned}$$

* In the most unfavourable case employed in the present research an error of about one part in 10,000 is involved in this assumption.

Hence $H^2 = kqV^2hp \cdot \tanh^2 al$.

$$\therefore k = \frac{H^2}{pqhV^2} \cdot \coth^2 al, \quad \dots \dots \dots (6)$$

which becomes, when l is large,

$$k = \frac{H^2}{pqhV^2} \cdot \dots \dots \dots (7)$$

IV. Experiments to Test the Applicability of the Formulæ.

(a) In the equation $k = \frac{H^2}{pqhV^2} \coth^2 al$,

if k , p , q , h and l are constant at a given temperature of the enclosure, H/V should remain constant; i.e.,

$$\frac{CE - CE'}{Pt_1 - Pt_2} \text{ should be constant.}$$

Many experiments with various wires were carried out to verify this, and provided the excess of temperature V of the hot end of the wire was made not greater than 10°C . or 12°C ., the results were exceedingly good. It would appear, however, that for greater temperature differences the value of h is not quite constant, but tends to increase slightly with the temperature. Some results for a eureka wire are given below :

November 13th, 1913.—Eureka Wire.

Diameter, 0.0995 cm. ; length, 40 cm.

Temperature of enclosure, 15.85°C .

In arbitrary units,

$$(1) \frac{CE - CE'}{Pt_1 - Pt_2} = \frac{6,817 \times 234.73 - 6,643 \times 228.78}{13.0260 - 12.8372},$$

= 425 for temperature difference 5.73°C .

$$(2) \frac{CE - CE'}{Pt_1 - Pt_2} = \frac{8,692 \times 297.1 - 8,465 \times 289.5}{13.1425 - 12.8320},$$

= 424 for temperature difference 8.96°C .

Similarly, for differences of temperature of 9.952°C . and 15.680°C ., the quotients came out as 425 and 428 respectively.

(b) Again, from equation (6), it appears that for a given temperature difference, V , the amount of heat H received at the hot end of a given wire varies in such a way as to keep $H \coth al$ constant. This, again, was tested, and the results were entirely satisfactory.

Below are given details of an experiment made on a copper wire (commercially pure), the length being varied from 45 cm. down to 5 cm.

December 9th and 10th, 1913.—Copper Wire.

Diameter, 0.1024 cm. ; k , 0.75 ; $\alpha = \sqrt{\frac{hp}{qk}} = 0.1665$.

Temperature, 17°C. (Arbitrary units are given.)

1.	$l =$	5 cm.	10 cm.	20 cm.	30 cm.	45 cm.
2	$\coth al \propto$	1.468	1.074	1.0025	1.000009	1
3	$\therefore H \propto$	$\frac{1}{1.468}$	$\frac{1}{1.074}$	$\frac{1}{1.0025}$	$\frac{1}{1.000009}$	1
4	Experimental $H =$	$\frac{1}{1.458}$	$\frac{1}{1.075}$	1	1	1

The third line in the table gives the relative values of H as calculated from the formula. The agreement of the fourth line (experimental determinations of H) with the third is exceedingly good. In the case of lengths of wire from 20 cm. to 45 cm. no difference whatever could be detected in the amount of heat H flowing into the wire for a given temperature difference V . For a length 10 cm. experiment gave the result correct to one part in 1,000 ; and for a length only 5 cm. the error was one part in 100, a result which must be considered very satisfactory. These results indicate that the thermal conductivity can be measured accurately with quite small lengths of wire, especially if the substance has a low conductivity. In the case of platinum, for example, if a length of only 5 cm. were employed, the diameter being only 1 mm., the factor $\coth^2 al$ is 1.1025. For a length 10 cm. it is only 1.0056.

The temperatures at various points along a long uniform rod heated at one end in an enclosure at constant temperature are readily calculated from equation (5)—

$$v = Ve^{-\alpha x}.$$

In the case of a silver wire of 1 mm. diameter, at tempera-
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ture 17°C. , $k=1$, $r=0.05$, $h=0.000533$, $a=0.1456$, we obtain :—

Distance from hot end.	Value of v .
x cm.	$Ve^{-0.1456x}$
5 "	$V \times 0.491$
10 "	$V \times 0.236$
15 "	$V \times 0.114$
20 "	$V \times 0.055$
30 "	$V \times 0.013$
40 "	$V \times 0.003$
50 "	$V \times 0.0007$

Again, the heat H lost from the first l cm. of wire is given by

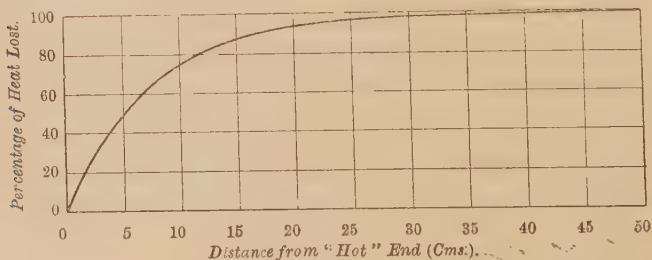
$$H = \int_0^l 2\pi rhv dx = C(1 - e^{-al}),$$

where C is put for the constant quantity $\frac{2\pi rhV}{a}$.

In the case of the silver wire above we have :—

Distance from hot end.	Heat lost.
l cm.	$C(1 - e^{-al})$
5 "	$C \times 0.5094$
10 "	$C \times 0.764$
15 "	$C \times 0.886$
20 "	$C \times 0.945$
30 "	$C \times 0.987$
40 "	$C \times 0.997$
50 "	$C \times 0.999$

The curve given below illustrates these results graphically. It is evident from the figure that in the case of a thin wire most of the heat given to the hot end of the specimen is lost from the first few centimetres.



(c) The resistance at the joint between the wire and the copper block into which it fits was investigated, and the con-

sequent fall of temperature there measured in the following way* :—

A "double joint," of precisely the same kind as employed in the main experiments, was constructed as shown in Fig. 4. A brass wire, RR, of rather larger diameter than the other specimens used, because of the difficulty of accurately boring the thinner wires, was fitted into the conical space ABCD. A hollow cone of thin brass (shaded in the figure) could also be inserted into this space. The wire then fitted into the hollow cone in exactly the same way as into the copper block. The two joints were, therefore, of precisely the same pattern and size.

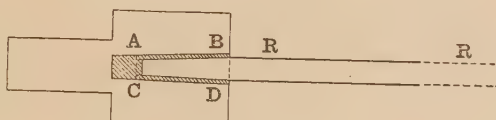


FIG. 4.

Determinations were made of the heat given to the wire :—

1. With the hollow cone in place.
2. The hollow cone being removed.

With the same notation as before, if the heat given to the rod

in case (1) is
$$H = \frac{CE - C'E'}{W^2 J},$$

and in case (2) is
$$H_1 = \frac{C_1 E_1 - C'E'}{W^2 J},$$

then
$$\frac{H}{H_1} = \frac{CE - C'E'}{C_1 E_1 - C'E'} \quad \dots \dots \dots (A)$$

Also
$$k = \frac{H^2}{pqhV^2} \coth^2 al.$$

Hence, for a constant temperature difference, V , (k , p , q , h , α and l being also invariable), H is proportional to V .

Hence, from equation (A),

$$\frac{V}{V_1} = \frac{H}{H_1} = \frac{CE - C'E'}{C_1 E_1 - C'E'} \quad \dots \dots \dots (B)$$

Measurements were made at laboratory temperatures and at 100°C ., and various temperature differences, V (given, as

* The method here described was suggested by Prof. C. H. Lees, and the apparatus was carefully made for me by G. F. Goodchild, Esq., M.A.

usual, by the simultaneous readings of the two platinum thermometers), were tried. It was found that the percentage fall of temperature was the same, within the errors of experiment, for all values of V , both at 100°C . and at the temperature of the laboratory. The constancy of the results of the experiments recorded in section IV. (a) also verifies this conclusion. Assuming that the "double joint" has the effect of doubling this fall of temperature, the value of the latter is given by $V - V_1$.

In the tables below are recorded two of the experiments, the first at about 17°C ., the second at 100°C .

I. Radius of wire, 0.15 cm. Length, 35 cm. Temperature, 17°C . Temperature difference, (V), 12.47°C .		
Wire in place without cone. $C=9,283$ $E=399.0$	Wire in, with cone. $C_1=9,254$ $E_1=396.9$	Wire out. $C'=7,437$ $E'=318.8$

$$\text{This gives } \frac{V}{V_1} = \frac{H}{H_1} = \frac{CE - C'E'}{C_1E_1 - C'E'} = 1.023.$$

II. Temperature, 100°C . Temperature difference (V), 11.32°C .		
$C=10,271$ $E=361.2$	$C_1=10,233$ $E_1=359.7$	$C'=8,117$ $E'=285.6$

$$\text{Here } \frac{V}{V_1} = \frac{CE - C'E'}{C_1E_1 - C'E'} = 1.022.$$

The mean value of V/V_1 , as the result of several concordant experiments, was 1.025. This indicates a fall of temperature of $2\frac{1}{2}$ per cent. of the measured temperature difference V . The values of V given by the thermometers are therefore higher by $2\frac{1}{2}$ per cent. than the actual differences of temperature between the hot end of the wire and the enclosure. In the results given in Table I., column X., the necessary corrections in the values of V have been made.

V. Determination of h .

As the formula for the thermal conductivity k involves a knowledge of " h ," the amount of heat lost by convection and radiation per second from 1 sq. cm. of surface of the wire when its temperature is 1°C . above that of the enclosure, careful experiments were made for the purpose of determining this quantity at the temperatures employed—viz., 17°C . and 100°C . It has been frequently assumed that the quantity h is not

dependent on the shape or size of the body from which heat is lost. In the present research most of the wires employed were of circular section, and about 1 mm. diameter. Some, however, had a diameter of only 0.5 mm. and others were of square section. It was of importance, therefore, to determine the quantity h for wires of sections corresponding to all those employed. Nickel and platinum wires were experimented upon, and gave practically identical results. The diameter of the wires (between the limits employed—0.5 to 1 mm.), seemed to have very little influence on the value of h . This is in agreement with the experiments of Ayrton and Kilgour,* whose curve connecting “emissivity” and

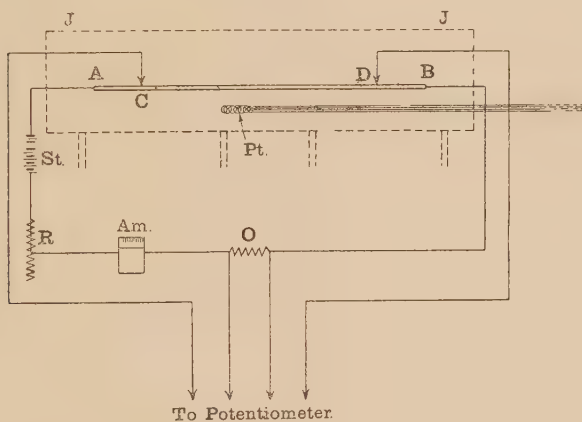


FIG. 5.

“radius of wire” becomes practically parallel to the latter axis when the diameter of the wire is 14 mils (0.356 mm.) or more.

Thin copper wire, gauge 30, was soldered to the extreme ends of the platinum wire (of length 40 cm.), and two wires of very thin platinum, or in some cases, platinoid, were welded or soldered with very great care—and little solder—to two points about 30 cm. apart. The latter wires served as potential leads. The wire was then placed in the middle of the water jacket, just as in the main experiment; a platinum thermometer also resting within the enclosure just opposite the middle of the wire (*see* Fig. 5).

* Ayrton and Kilgour, *Phil. Trans.*, clxxxiii., Part 1, p. 371, 1892.

Va. *Measurement of c , the Temperature Coefficient of the Wires.*

A small current, C , measured roughly by an ammeter, Am , and accurately by means of a standard 1 ohm coil, and Rayleigh potentiometer, was sent through the platinum wire AB . The P.D. E between the points C and D was measured in the same way. $R(=E/C)$ then gives the resistance of the wire between C and D at the temperature of the enclosure (or slightly higher, unless the current is very small), as given by the platinum thermometer. The temperature was then raised to about 100°C . by passing steam through the jacket, and the resistance was again measured as before. From the two values of the resistance, and those of the temperature, the temperature coefficient c was calculated. The constancy of the value of c obtained when different strengths of current were used afforded evidence that the heat lost down the potential leads was too small to be taken into account.

Special experiments were done to find the actual temperature coefficients between the limits (1) 17°C . to about 27°C . ; (2) 100°C . to about 110°C . For this purpose the platinum wire was placed within a copper cylinder 60 cm. long (replacing the water-jacket mentioned above), round which was wound as evenly as possible a coil of manganin wire. The temperature, as shown by the platinum thermometer inside the cylinder, was then raised to various values up to 120°C . by sending an electric current through the manganin wire till the required temperature was attained. When this was quite steady the resistance of the platinum was measured as before. No difference in the temperature coefficients could be detected at temperatures within the range of the present series of experiments.

The mean value obtained for c for a platinum wire of diameter 1 mm., as the result of several concordant experiments, was 0.00386, with an agreement to within about 1 part in 300. Below is recorded one of the experiments :—

September 20, 1913.—Platinum Wire.

Circular section, diameter 0.1006 cm. Distance between potential leads, 28.95 cm.

1. Water in jacket.	2 Steam through jacket.
Pt = 16.446°C .	$99.603^{\circ}\text{C} = \text{Pt}$.
E = 208.88	271.37 = E.
C = 5352	5341 = C.
R = 0.03903 ohm.	0.05081 = R.

* E and C are readings on the Rayleigh potentiometer, in ohms, corresponding to the P.D. between the points C and D on the wire, and the current through the wire respectively. The resistance R is obtained by the division of E by C.

$$\begin{aligned} \text{Hence} \quad & \frac{5081}{3903} = \frac{1+99.603C}{1+16.446C}, \\ \text{and} \quad & C = 0.00386. \end{aligned}$$

Vb. *Measurement of "h" for Platinum Wire of Diameter 1 mm.*

A comparatively large current, C (about 2 to 2.5 amperes), was now sent through the platinum wire, placed in precisely the same position as in the main experiment, described in Section IIa. The P.D. E and the amount of heat Q given to the part CD were also determined.

$$Q = CE/J. \quad \dots \dots \dots (I)$$

The temperature (t') of the enclosure was taken at the same time by the platinum thermometer. A small current (C') was now sent through the wire, and the P.D. (E') measured between C and D, the wire being first allowed to cool to a temperature that would be equal to (or very slightly higher than) that of the enclosure. The change of resistance of the wire is, of course, given by the expression $E/C - E'/C'$.

With a knowledge of the temperature coefficient c of the wire the excess of temperature of the latter over that of the enclosure can be accurately determined. The amount of heat given to that part of the wire between C and D, and thence to the enclosure, is given by

$$(CE - C'E')/J. \quad \dots \dots \dots (II.)$$

Hence if s is the surface area of the wire between C and D, and $(t - t')$ the excess of temperature of the wire over that of the enclosure, we have

$$h = \frac{CE - C'E'}{J(t - t')s}. \quad \dots \dots \dots (III.)$$

Experiments were conducted at temperatures 17°C. and 100°C. The mean results obtained for this particular wire, from a large number of experiments, were

$$h = 0.000533 \text{ at } 17^\circ\text{C.}; \text{ and } 0.000634 \text{ at } 100^\circ\text{C.}$$

It is assumed that h has the same value for all the metals (of identical cross-section) employed in the present research. In this connection it may be noted that the loss of heat is almost entirely due to convection, so that the difference in emissivity of the various metals (which is not likely to be great, however), would scarcely affect the value of h . It has been already stated that nickel and platinum, at any rate, give

identical values for this quantity. Below are given examples of the determination of h , together with the method of reducing the results.

September 23rd, 1913.—Platinum Wire.

Circular section, diameter 1 mm. Surface area of platinum between potential leads, 9.149 sq. cms.

I.—Large current.	II.—Small current.
Reading of ammeter 2.05 amps.	0.125 amp
Pt. 17.694°C.	17.700°C.
E 202.74	37.343
C 5178	981.41
W 2576	7821
R 0.03915 ohm.	0.03805 ohm.

If t =temperature of platinum when current 2.05 amperes flows through it we have, $\frac{R_t}{R_{17.7}} = \frac{3.915}{3.805} = \frac{1+0.00386 \times t}{1+0.00386 \times 17.7}$, from which $t=25.701^\circ\text{C.}$, and excess of temperature of wire over that of enclosure= 8.007°C.

$$\text{Also } Q = \left\{ \frac{5,178 \times 202.74}{(2,576)^2} - \frac{981.41 \times 37.343}{(7,821)^2} \right\} \times 0.2479^*$$

$$\therefore Q = 0.03904 \text{ calories.}$$

$$\therefore h = \frac{Q}{s(t-t')} = 0.000533.$$

In precisely the same way the value of h at 100°C. was found to be 0.000634 for the same wire.

The corresponding results for a platinum wire of square cross-section were as follows :—

April 24th, 1914.—Platinum Wire.

Square section; diameter, 0.795 mm. Temperature coefficient of resistance, 0.00358. Surface area of wire between potential leads, 8.43 sq. cms.

I.—Large current.	II.—Small current
Reading of ammeter, 1.72 amp.	0.22 amp.
Pt. 16.851°C.	16.851°C.
E 320.73	39.72
C 6587	839.0
W 3814	3814
R 0.04869 ohm.	0.04735 ohm.

$$*0.2479 = \frac{(\text{E.M.F. Weston Cell})^2}{J} = \frac{(1.0186)^2}{4.185}$$

Excess of temperature of wire over that of enclosure
 $= 8.412^{\circ}\text{C}$. $Q = 0.03541$ calories.

$$\therefore h = \frac{0.03541}{8.412 \times 8.43} = 0.000499 \text{ at } 17^{\circ}\text{C}.$$

Similarly the value of h at 100°C . was 0.000594.

In connection with these results, an interesting Paper by Dr. Alexander Russell,* which throws valuable light on the variation of h under different conditions, is worthy of mention. It appears that h is not under all conditions proportional to the difference of temperature, θ , between the hot body and its surroundings. For example, L. Lorenz† obtained, mathematically, the result, that in the case of a heated strip, $h \propto \theta^{\frac{2}{3}}$. On the other hand, Boussinesq‡ deduced that the convection of heat by a stream of liquid from a cylinder maintained at a constant temperature is given by $h \propto \theta$. This result is confirmed by results obtained by P. Compan,§ and also by Kennelly,|| in the case of the cooling of cylindrical wires.

Again, as Russell remarks, Newton's law ($h \propto \theta$) leads to results in several practical applications that are found to be in close accordance with experiment.

The results of the present experiments, *e.g.*, those given in section IV. (a), indicate that with this particular arrangement of apparatus, Newton's law is strictly applicable, at any rate for values of θ up to 10°C . or 12°C .

VI. *Determination of Thermal Conductivity k .*

The following is a full account of the determination of the remaining quantities necessary for obtaining " k ," together with the method of reducing the results, the particular case taken being that of pure platinum at the temperature of the laboratory. For temperatures 100°C . the only difference in procedure was to send steam through the outer jacket.

* A. Russell, "Proc." Phys. Soc., XXII., p. 432, 1909.

† L. Lorenz, "Ann. der Physik," XIII., p. 582, 1881.

‡ Boussinesq, "Comp. Rend.," CXXXIII., p. 257.

§ P. Compan, "Ann. de Chim. et Phys.," XXVI., p. 488, 1902.

|| Kennelly, "Amer. Inst. Elec. Engin. Proc.," July, 1909.

November 4th, 1913.—Platinum.

Radius of wire, 0.0503 cm. $\therefore p.q.h. = 2\pi^2 r^2 h = 1339 \times 10^{-9}$. Length of specimen, 35.1 cms.

Platinum out.	Platinum in.
$Pt_1 = Pt_2 = V = 12.177^\circ C.$ $E' = 7618.$ $C' = 310.8.$ $W = 3839.$	$Pt_1 - Pt$ maintained $12.177^\circ C.$ $E = 8140.$ $C = 331.7.$ $W = 3839.$
E.M.F. Weston = 1.0186 volt.	

Hence, true current $= C/W \times 1.0186$.

True E.M.F. $= E/W \times 1.0186$.

and heat given $= \frac{EC}{JW^2} \times (1.0186)^2$, with wire in.

and $= \frac{E'C'}{JW^2} \times (1.0186)^2$, with wire out.

H (measured in calories) $= \frac{CE - C'E'}{JW^2} \times (1.0186)^2$.

$\therefore H^2 = 508.5 \times 10^{-6} \times 6.146 \times 10^{-2}$.

(N.B.— $6.146 \times 10^{-2} = \left[\frac{1.0186^2}{J} \right]^2$).

$H^2 = 31.26 \times 10^{-6}$.

$V^2 = (11.880)^2 = 141.13$.

$\therefore k = \frac{H^2}{pqhV^2} = 0.165$.

In Table I. the thermal conductivities of all the metals examined are tabulated, together with the principal measurements involved. The figures given are the corrected ones, the principal corrections being those due to (1) the calibration of the coils of the resistance boxes used in the Rayleigh potentiometer, (2) the standardisation of the 1-ohm resistance coil and of the Weston cells, (3) the fall of temperature at the junction of the wire and the copper block. In the calculation of V in column X., where V is the difference of temperature as given by the readings of the two platinum thermometers Pt_1 and Pt_2 , a curve was drawn on a large scale connecting the simultaneous readings of the two thermometers at various temperatures from $0^\circ C.$ to $100^\circ C.$ It was therefore a simple matter to convert the readings of Pt_2 to terms of Pt_1 , obtain the value of V in terms of platinum, and thence reduce to gas.

TABLE I.—*Thermal Conductivities (k) at Temperature*

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.
Date	Metal.	Diameter (cms.)	$pqh \times 10^9$	E (ohms).	C (ohms).	E' (ohms).	C' (ohms).	W (ohms).
Nov. 4, 1913	Platinum ...	0.1006 {	1,339	8,140	331.7	7,618	310.8	3,839
" 10 "			1,593	7,502	254.1	7,125	241.3	2,589
Nov. 21 "	Platinum— 10% iridium	0.1025 {	1,416	5,353	219.9	5,116	210.1	2,587
Dec. 2 "			1,684	5,810	198.6	5,600	191.5	2,584
Dec. 11 "	Platinum— 15% iridium	0.1025 {	1,416	4,815	198.3	4,626	190.5	2,584
" 17 "			1,684	5,789	197.8	5,604	191.4	2,581
Jan. 7, 1914	Platinum— 20% iridium	0.1006 {	1,339	7,077	293.9	6,844	284.5	3,848
" 8 "			1,593	8,692	297.2	8,465	289.5	3,849
Jan. 29 "	Platinum— 10% rhodium	0.1018 {	1,387	7,726	316.0	7,391	302.3	3,844
" 29 "			1,650	9,276	316.3	8,947	305.4	3,844
Mar. 9 "	Iridium ...	0.1030 (sq.section)	2,181	7,207	311.1	6,670	287.9	3,840
" 10 "			2,596	8,734	310.8	8,212	292.6	3,840
Mar. 12 "	Rhodium	0.1030 (sq.section)	2,181	7,671	327.1	6,933	297.3	3,838
" 14 "			2,596	8,620	305.7	8,020	284.5	3,836
Mar. 4 "	Gold	0.1014 {	1,371	7,332	317.5	6,433	278.7	3,850
" 9 "			1,630	8,717	311.0	7,876	281.0	3,840
Nov. 13, 1913	Eureka (constantan)	0.0995 {	1,295	5,256	215.0	5,070	207.0	2,591
" 13 "			1,541	6,080	207.9	5,894	201.4	2,590
Jan. 1, 1914	Tungsten ("pladuram")	0.0600 {	284	7,294	309.4	6,926	293.6	3,843
Dec. 22, 1913			338	9,382	319.6	9,003	307.0	3,848
Jan. 13, 1914	Palladium (commercial)	0.1010 {	1,355	8,046	330.7	7,639	314.0	3,847
" 12 "			1,612	9,379	319.7	9,003	307.1	3,848
Jan. 19 "	Palladium (pure)	0.0905 {	975	7,611	315.8	7,213	299.4	3,847
" 17 "			1,160	8,671	297.0	8,321	284.9	3,847
Feb. 11 "	Tantalum ...	0.0475 {	141	6,980	299.6	6,840	293.6	3,837
" 12 "			168	8,597	304.2	8,464	299.6	3,837
Feb. 17 "	Molybdenum	0.0520 {	281	7,263	310.06	6,940	295.9	3,837
" 18 "			335	8,826	312.25	8,526	301.6	3,837
Mar. 19 "	Graphite.....	0.183 {	8,059	7,027	304.0	6,500	281.0	3,834
" 21 "			9,584	8,499	301.9	7,999	284.1	3,836

Col. IV.— p =perimeter; q =cross-sectional area of wire; h =heat lost per second per

Cols. V. to IX.—E, C, E', C' and W are readings on Rayleigh Potentiometer.

Col. X.—V=excess of temperature of hot end of wire over enclosure.

Col. XIV.—J. and D. refers to Jäger and Diesselhorst's Paper, *loc. cit.*

17° and 100°C.

X.	XI.	XII.	XIII.	XIV.
V° Cent.	<i>l</i> (cms.)	Temp. of enclosure (Cent.)	$\frac{1}{2}$ (Cals. per cm.-deg.C.)	Remarks.
11-880 13-274	} 35.2 {	17 100	0.165 0.170	} J. and D., 0.166 and 0.173
11-675 8-557	} 35.1 {	17 100	0.074 0.075
9-612 8-534	} 35.2 {	17 100	0.056 0.059
9-326 8-608	} 35.5 {	17 100	0.042 0.042
11-018 9-765	} 35.1 {	17 100	0.072 0.073
9-740 8-833	} 10.0 {	17 100	0.141 0.135	} Spec. gravity, 22.33.
10-880 8-430	} 10.0 {	17 100	0.210 0.192	} Spec. gravity, 12.505.
9-103 7-832	} 35.2 {	17 100	0.705 0.701	Spec. gravity, 19.49. J. and D., 0.700 and 0.702.
11-385 9-709	} 40.0 {	17 100	0.053 0.056	} J. and D., 0.054 and 0.064.
10-184 9-824	} 28.5 {	17 100	0.476 0.472	Kindly lent by Messrs. Isenthal and Co., Neasden.
11-869 9-729	} 35.1 {	17 100	0.101 0.100
10-883 8-399	} 35.2 {	17 100	0.144 0.143	} J. and D., 0.168 and 0.182.
10-302 9-083	} 48.0 {	17 100	0.130 0.129	} Spec. gravity, 16.67.
10-739 9-297	} 45.0 {	17 100	0.346 0.333	} Spec. gravity, 9.933.
9-489 8-154	} 10.4 {	17 100	0.037 0.038	" Graphite " from a " Kohinoor" lead pencil (6H): sp. gr., 2.11.

square centimetre of surface per 1°C. excess of temperature.

temperatures. This comparison of the readings of the two thermometers was frequently made during the course of the research, and it was satisfactory to find no appreciable alteration in their relative resistances within the range of temperature employed. The wire used for the thermometer Pt_1 was from the same reel as one whose δ coefficient had previously been obtained, and had the value of 0.000152 in the formula,

$$t - Pt = \delta t(t - 100),$$

where t is the gas temperature, Pt the platinum. The fixed points taken for the determination of δ were, as usual, those of ice, steam and boiling sulphur.

In cases where results by other investigators are available they are given in column XIV. for the sake of comparison. As a whole, the results of the present Paper are in good agreement. In the case of the majority of the metals experimented upon, however, no previous determinations of the thermal conductivity appear to have been made.

VII. *Electrical Resistances of Wires.*

The electrical resistances of the same wires (including also graphite) were determined at laboratory temperatures and at about 100°C . in the following way:—

The wire was placed on two copper knife-edges, EE (Fig. 6), and fastened down firmly with silk thread, TT . Two thin silk-covered copper wires (gauge 30) were soldered to the knife-edges, which were kept at a fixed measured distance apart by

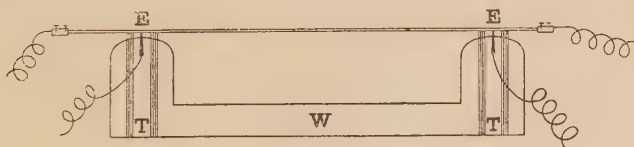


FIG. 6.

being inserted in parallel vertical grooves made in a piece of hard wood, W , shaped as in the figure.

A current, C , from two storage cells was sent through the specimen, and the E.M.F. (E) between the knife-edges measured, as in the heat experiments, by means of a potentiometer and a Weston cell, the current being determined also in a similar way. The metals were placed in the same enclosure as in the heat experiments, and measurements made at about

17°C. and 100°C., the temperatures being given by a platinum thermometer placed midway between EE. The same current, approximately, was used at both temperatures and was too small to cause an appreciable rise of temperature of the wire. Details are given below of an experiment on platinum-iridium (10 per cent. iridium), and Table II. gives the measurements similarly obtained with the remaining wires at both temperatures.

November 25th, 1913.—Platinum-Iridium (10 per cent.).

Diameter, 0.1025 cm.; distance l between potential leads, 28.14 cm.

Temperature, 16.53°C.

Current C (in terms of potentiometer and Leclanché cells), 568.8 ohms.

E.M.F. E (in terms of potentiometer), 50.24 ohms.

Resistance $R = E/C = 0.08845$ ohm.

Specific resistance

$$(\rho) = \frac{R \cdot \pi r^2}{l} = \frac{0.08845 \pi (0.05125)^2}{28.14} = 25.88 \times 10^{-6} \text{ ohm per cm. c'be.}$$

The same result for the value of R to four significant figures could nearly always be obtained, using different values of the current and of the E.M.F. This afforded satisfactory evidence of the correctness of the calibration of the coils or the resistance boxes used in the Rayleigh potentiometer. A 1/100th ohm resistance box was included in the latter.

In Table II. the "length" given in column IV. is, of course, the distance apart of the knife-edges between which the P.D. E was measured. The resistances tabulated in column VII. are then obtained from the quotient E/C . Columns IX. and X. are obtained by a slight extrapolation of the results from column VIII. The comparatively high values obtained for α_{100} in the case of platinum, tungsten, palladium, tantalum and molybdenum are evidences of the purity of those metals. All the metals employed (except the tungsten) were supplied by Messrs. Johnson, Matthey & Co., and were of the highest purity obtainable.

VIII. *Discussion of the Results.*

In order to enable the results of the present Paper to be compared with the electronic theories the values of k/KT are given in Table III. k is the thermal conductivity, K

TABLE II.—Electrical Resistances of Specimens (ohms per centimetre cube).

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.	XIII
Date.	Metal.	Temp. Cent.	Diameter (cms.)	Length (cms.)	Current (C.)	E.M.F. (E.)	Resistance (ohms).	Spec. res. ("ρ" × 10 ⁶)	ρ ₀ × 10 ⁶	ρ ₁₀₀ × 10 ⁶	α ₁₀₀	Remarks.
Nov. 25, 1913 " 25 "	Platinum ...	{ 17.70 99.28 }	{ 0.1006 }	28.95 {	{ 981.41 932.31 }	{ 37.343 46.134 }	{ 0.03805 0.04948 }	{ 10.45 13.59 }	9.766	13.613	0.00394	{ D. and F., 10.96 (at 0°); 14.86 (at 100°C.) J. and D., 10.82 (at 0°); 15.11 (at 100°C.)
Nov. 24 " 25 "	Platinum— 10% iridium	{ 16.53 99.12 }	{ 0.1025 }	28.14 {	{ 568.08 567.75 }	{ 50.24 54.91 }	{ 0.08845 0.09671 }	{ 25.88 28.31 }	25.397	28.323	0.00115
Jan. 1, 1914 " 2 "	Platinum— 15% iridium	{ 3.84 99.08 }	{ 0.1025 }	28.14 {	{ 554.85 554.39 }	{ 54.515 59.29 }	{ 0.09825 0.1069 }	{ 28.75 31.28 }	28.65	31.31	0.000928
Jan. 7 " 8 "	Platinum— 20% iridium	{ 16.94 99.25 }	{ 0.1006 }	28.14 {	{ 580.31 600.47 }	{ 68.97 75.66 }	{ 0.11886 0.12600 }	{ 33.58 35.59 }	33.16	35.61	0.000739	D. and F., 31.54 (at 0°); 32.35 (at 100°C.)
Jan. 22 " 22 "	Platinum— 10% rhodium	{ 12.07 99.00 }	{ 0.1018 }	28.14 {	{ 754.55 754.89 }	{ 59.53 66.45 }	{ 0.07889 0.08802 }	{ 22.82 25.46 }	22.45	25.49	0.00135	D. and F., α=0.00143.
Mar. 23 " 19 "	Iridium.....	{ 12.11 97.20 }	{ 0.1030 (sq. section) }	7.45 {	{ 906.51 1332.5 }	{ 5.398 9.840 }	{ 0.005955 0.007384 }	{ 8.480 10.521 }	8.190	10.59	0.00293	B. and H., 6.10 (at 0°); 8.33 (at 100°C.)
Mar. 24 " 23 "	Rhodium ...	{ 14.93 97.60 }	{ 0.1030 (sq. section) }	7.45 {	{ 1703.2 1331.9 }	{ 6.005 6.276 }	{ 0.003525 0.004712 }	{ 5.020 6.710 }	4.811	6.211	0.00291	B. and H., 4.70 (at 0°C.); 6.60 (at 100°C.)
Feb. 3 " 4 "	Gold	{ 16.94 99.09 }	{ 0.1004 }	28.14 {	{ 2326.5 2323.7 }	{ 19.965 25.447 }	{ 0.008581 0.01951 }	{ 2.405 3.069 }	2.268	3.075	0.00356	{ J. and D., 2.422 (at 18°); 3.112 (at 100°C.) D. and F., 2.198 (at 0°); 2.941 (at 90.4°C.) N., 2.247 (at 0°); 3.106 (at 100°C.)
Nov. 13, 1913 " 17 "	Eureka (Constantan)	{ 6.01 99.49 }	{ 0.0995 }	28.14 {	{ 562.15 561.85 }	{ 93.44 93.27 }	{ 0.16613 0.16601 }	{ 45.90 45.87 }	45.90	45.87	0.0057	{ N., 44.05 (at 0°); 44.64 (at 100°C.) J. and D., 49.01 (at 0°); 49.09 (at 100°C.) S., α=0.008 (at 12.5°); 0.052 (at 25°); 0.033 (at 100°C.)
Jan. 2, 1914 " 3 "	Tungsten ("Pladuram")	{ 7.00 99.00 }	{ 0.0600 }	26.58 {	{ 569.0 568.15 }	{ 28.73 40.26 }	{ 0.05049 0.07087 }	{ 5.371 7.539 }	5.206	7.562	0.00453
Jan. 10 " 10 "	Palladium (commercial)	{ 4.96 99.57 }	{ 0.1010 }	28.14 {	{ 597.35 596.73 }	{ 37.37 38.84 }	{ 0.06257 0.06509 }	{ 17.815 18.532 }	17.78	18.54	0.000427
Jan. 19 " 21 "	Palladium (pure)	{ 13.26 99.14 }	{ 0.0905 }	28.14 {	{ 377.11 378.05 }	{ 17.050 22.324 }	{ 0.04521 0.05905 }	{ 10.334 13.497 }	9.845	13.529	0.00374	{ D. and F., 10.214 (at 0°); 13.793 (at 98.5°C.); α=0.00354 K., 10.649 (at 0°); α=0.00302 J. and D., 10.718 (at 18°); 13.755 (at 100°C.)
Feb. 4 " 4 "	Tantalum ...	{ 13.47 98.70 }	{ 0.0475 }	28.14 {	{ 454.35 453.09 }	{ 108.84 137.57 }	{ 0.2396 0.3026 }	{ 15.088 19.118 }	14.452	19.178	0.00327	V.P., 14.60 (at 0°C. ?); α=0.0033.
Feb. 19 " 20 "	Molybdenum	{ 15.88 97.85 }	{ 0.0520 (sq. section) }	28.14 {	{ 405.52 456.23 }	{ 26.317 40.159 }	{ 0.06490 0.08802 }	{ 6.236 8.458 }	5.806	8.516	0.00467	S., α=0.0034.
Mar. 27 " 30 "	Graphite ...	{ 14.25 98.35 }	{ 0.183 }	7.45 {	{ 1372.8 1537.0 }	{ 250.21 298.85 }	{ 5.481 5.143 }	{ 19300 18200 }	19500	18100	0.00073	F.S., 14200 at 0°C.

Col. XIII.—D. and F. refers to Dewar and Fleming, "Phil. Mag." 5, 36, 271, 1893; and Roy. Inst. Gr. Brit., June 5, 1896. J. and D., Jäger and Diesselhorst, *loc. cit.* N., Niccolai, *Lineei*, end., 5, 16. B. and H., Bronewski and Hackspill, C.R., 153, 814, 1911. K., Knott, "Proc." Roy. Soc. Edinburgh, 18, 303, 1891. V. P., V. Pirani, Z. S. Elch., 11, 45, 1905. F. S., F. Streintz, Z. S. Elch., 11, 273, 1905. S., Somerville, "Phys. Rev.", 31, 261, 1910.



TABLE III.—*Values of k/KT^* at 273° and at 373° (A°_s Temp.)*

I.	II.	III.	IV.	V.	VI.
Metal.	Thermal conductivity k .	Electrical conductivity $K \times 10^{-4}$	Abs. temp. T.	$KT \times 10^{-6}$	$k/KT \times 10^8$
Platinum	$\begin{cases} 0.691 \\ 0.711 \end{cases}$	$\begin{cases} 10.24 \\ 7.35 \end{cases}$	$\begin{cases} 273 \\ 373 \end{cases}$	$\begin{cases} 27.95 \\ 27.40 \end{cases}$	$\begin{cases} 2.47 \\ 2.59 \end{cases}$
Platinum— 10% iridium	$\begin{cases} 0.310 \\ 0.314 \end{cases}$	$\begin{cases} 3.98 \\ 3.53 \end{cases}$	$\begin{cases} 273 \\ 373 \end{cases}$	$\begin{cases} 10.86 \\ 13.17 \end{cases}$	$\begin{cases} 2.85 \\ 2.38 \end{cases}$
Platinum— 15% iridium	$\begin{cases} 0.234 \\ 0.247 \end{cases}$	$\begin{cases} 3.49 \\ 3.19 \end{cases}$	$\begin{cases} 273 \\ 373 \end{cases}$	$\begin{cases} 9.53 \\ 11.90 \end{cases}$	$\begin{cases} 2.45 \\ 2.08 \end{cases}$
Platinum— 20% iridium	$\begin{cases} 0.176 \\ 0.176 \end{cases}$	$\begin{cases} 3.02 \\ 2.81 \end{cases}$	$\begin{cases} 273 \\ 373 \end{cases}$	$\begin{cases} 8.23 \\ 10.48 \end{cases}$	$\begin{cases} 2.14 \\ 1.68 \end{cases}$
Platinum— 10% rhodium	$\begin{cases} 0.301 \\ 0.306 \end{cases}$	$\begin{cases} 4.46 \\ 3.92 \end{cases}$	$\begin{cases} 273 \\ 373 \end{cases}$	$\begin{cases} 12.16 \\ 14.63 \end{cases}$	$\begin{cases} 2.48 \\ 2.10 \end{cases}$
Iridium.....	$\begin{cases} 0.590 \\ 0.565 \end{cases}$	$\begin{cases} 12.21 \\ 9.44 \end{cases}$	$\begin{cases} 273 \\ 373 \end{cases}$	$\begin{cases} 33.33 \\ 35.22 \end{cases}$	$\begin{cases} 1.77 \\ 1.60 \end{cases}$
Rhodium	$\begin{cases} 0.879 \\ 0.804 \end{cases}$	$\begin{cases} 20.78 \\ 16.10 \end{cases}$	$\begin{cases} 273 \\ 373 \end{cases}$	$\begin{cases} 56.7 \\ 60.1 \end{cases}$	$\begin{cases} 1.55 \\ 1.33 \end{cases}$
Gold	$\begin{cases} 2.95 \\ 2.93 \end{cases}$	$\begin{cases} 44.09 \\ 32.52 \end{cases}$	$\begin{cases} 273 \\ 373 \end{cases}$	$\begin{cases} 120.4 \\ 121.3 \end{cases}$	$\begin{cases} 2.45 \\ 2.41 \end{cases}$
Eureka (Constantan)	$\begin{cases} 0.222 \\ 0.234 \end{cases}$	$\begin{cases} 2.179 \\ 2.180 \end{cases}$	$\begin{cases} 273 \\ 373 \end{cases}$	$\begin{cases} 59.5 \\ 81.3 \end{cases}$	$\begin{cases} 3.73 \\ 2.87 \end{cases}$
Tungsten ("Pladuram")	$\begin{cases} 1.99 \\ 1.97 \end{cases}$	$\begin{cases} 19.21 \\ 13.22 \end{cases}$	$\begin{cases} 273 \\ 373 \end{cases}$	$\begin{cases} 52.44 \\ 49.33 \end{cases}$	$\begin{cases} 3.79 \\ 3.99 \end{cases}$
Palladium (commercial)	$\begin{cases} 0.423 \\ 0.418 \end{cases}$	$\begin{cases} 5.62 \\ 5.39 \end{cases}$	$\begin{cases} 273 \\ 373 \end{cases}$	$\begin{cases} 15.34 \\ 20.10 \end{cases}$	$\begin{cases} 2.70 \\ 2.08 \end{cases}$
Palladium (pure)	$\begin{cases} 0.603 \\ 0.598 \end{cases}$	$\begin{cases} 10.16 \\ 7.39 \end{cases}$	$\begin{cases} 273 \\ 373 \end{cases}$	$\begin{cases} 27.74 \\ 27.56 \end{cases}$	$\begin{cases} 2.18 \\ 2.16 \end{cases}$
Tantalum.....	$\begin{cases} 0.544 \\ 0.540 \end{cases}$	$\begin{cases} 6.92 \\ 5.21 \end{cases}$	$\begin{cases} 273 \\ 373 \end{cases}$	$\begin{cases} 18.89 \\ 19.43 \end{cases}$	$\begin{cases} 2.88 \\ 2.78 \end{cases}$
Molybdenum	$\begin{cases} 1.45 \\ 1.39 \end{cases}$	$\begin{cases} 17.22 \\ 11.74 \end{cases}$	$\begin{cases} 273 \\ 373 \end{cases}$	$\begin{cases} 47.01 \\ 43.80 \end{cases}$	$\begin{cases} 3.08 \\ 3.17 \end{cases}$
Graphite	$\begin{cases} 0.155 \\ 0.159 \end{cases}$	$\begin{cases} 0.00513 \\ 0.00552 \end{cases}$	$\begin{cases} 273 \\ 373 \end{cases}$	$\begin{cases} 0.0140 \\ 0.0206 \end{cases}$	$\begin{cases} 1,110 \\ 770 \end{cases}$

* k = Thermal conductivity in watts per centimetre-degree Cent.
 K = Electrical conductivity in reciprocal ohms per centimetre-cube.
 T = Absolute gas temperature.

the electrical conductivity, and T the absolute temperature on the hydrogen scale. As a general rule, the values given in column VI. tend towards a common value somewhere about 2.5, though two or three of the metals, notably tungsten, rhodium and iridium, differ rather widely from this generalisation. Graphite is, of course, a very pronounced exception, for its thermal conductivity is very nearly as great as that of platinum 20 per cent. iridium, while its electrical conductivity is about 600 times less. Lees has observed a parallel case in the comparison of the conductivities of bismuth and quartz, the latter being one of the very best of the electrical insulators, while having at the same time a thermal conductivity greater than that of bismuth. The platinum-iridium series given in Table III. is interesting, as affording a good example of the influence of "impurities" on the thermal and electrical properties of a metal. Grüneisen * showed that addition of foreign substances reduces the electrical conductivity to a greater extent than the thermal—at any rate in cases where the amount of impurity is comparatively small. The following table is compiled from some of his measurements. Cu indicates pure copper, CuAs₁ copper with a small percentage of arsenic, CuAs₂ copper with a larger percentage of arsenic:—

Metal.	Electrical conductivity, $K \times 10^5$.	Thermal conductivity, λ .	$\lambda/K \times 10^{-1}$.
Cu	57.4	0.934	163
CuAs ₁	19.1	0.340	178
CuAs ₂	5.03	0.0995	198

Below is given a similar table derived from Table III. :—

Metal.	Electrical conductivity, K.	Thermal conductivity, λ .	λ/K
Platinum	10.24	0.691	675
Pt-10% Ir.	3.98	0.310	779
Pt-15% Ir.	3.49	0.234	670
Pt-20% Ir.	3.02	0.176	583
Iridium	12.21	0.594	487

As in Grüneisen's results for copper, the 10 per cent. alloy shows an increase in λ/k over that given by the pure platinum. As the "impurity" increases, however, the thermal conductivity decreases more rapidly than the electrical. The values

* E. Grüneisen, Ann. de Physik., 3, 1, pp. 43-74, Sept., 1900.

obtained for the thermal conductivity of two specimens of palladium are rather striking. A commercially "pure" specimen gave values of 0.101 and 0.100 at 0°C. and 100°C. respectively, while the pure metal gave 0.144 and 0.143 at corresponding temperatures. The presence of what was presumably quite a small amount of impurity reduced the conductivity by over 30 per cent.

Lees' observation that for pure metals the thermal conductivity decreases with rise of temperature, while for alloys it increases as the temperature rises, is, on the whole, well borne out in the present experiments. Platinum (as in Jäger and Diesselhorst's experiments) forms an exception to this rule. The changes of conductivity, however, are in many cases so slight as to be almost within the errors of experiment.

Perhaps the least satisfactory measurement of those involved is that of the radius of the specimens. With the wires employed, however, which had been used for no other purpose previously, there was surprisingly little variation in diameter, which could be measured correctly at any rate to one-third of 1 per cent. As the formulæ employed involve the cube of the radius this means a possible error of about 1 per cent.

IX. *Particulars of Instruments Employed.*

The *Micrometer Screw Gauge* used in the measurement of the diameters was one by the Brown & Sharpe Manufacturing Co. It was tested in the National Physical Laboratory, and certified to be correct (at 20°C.) to within 0.005 mm. at eight points from 0.5 mm. to 4.0 mm.

The *Standard Resistance Coil* was supplied by Messrs. J. J. Griffin, and was found to have a resistance of 0.99859 true ohm at 20°C., as compared with the N.P.L. standards. It was kept in paraffin oil, whose temperature during the research varied very little from 17°C.

Three *Standard Cadmium (Weston) Cells* were employed, which agreed with each other to within one or two parts in 10,000. One was made by myself according to specifications given by Mr. F. E. Smith,* of the National Physical Laboratory, and the other two were supplied by Messrs. J. J. Griffin. One of these cells was certified by the N.P.L. as having an E.M.F. of 1.0186 volt at 20°C.

The *Galvanometer*, which was used in connection with the

* F. E. Smith, "Phil. Trans.," A, 207, pp. 393-420, 1908.

Callendar-Griffiths' bridge for platinum temperature measurements, and also with the potentiometer, was of the Ayrton-Mather moving-coil pattern, by the Cambridge Scientific Instrument Co. Its resistance at 15°C. was 19.4 ohms, its period 8.6 seconds, and its deflection in millimetres at 1 metre for a current of 1 micro-ampere was 350.

The Callendar-Griffiths' Bridge, used in connection with the platinum thermometers, was carefully calibrated in the usual way. With its aid temperatures could be accurately measured within the ranges employed to 0.001°C.

The figures given in the various tables of the present Paper have all been corrected according to the specifications given above.

The research has been carried out at the Wandsworth Technical Institute, and my thanks are due to the Principal (Mr. G. F. Goodchild, M.A., B.Sc.) for the kind interest he has taken in the work; also to Mr. J. J. Risdon, of the firm of Messrs. Johnson, Matthey & Co., for his never failing courtesy in preparing and supplying the specimens used.

I am also very deeply indebted to Prof. C. H. Lees for valuable hints and criticisms during the course of the research.

ABSTRACT.

A new method of the "stationary temperature" type is employed for measuring the thermal conductivities of some of the rarer metals, including tantalum, molybdenum, rhodium, iridium and tungsten, at air temperatures and at 100°C.

It is shown that if a rod of metal is of length l , perimeter p , and cross-section q , then its thermal conductivity k is given by

$$k = \frac{H^2}{pqhV^2} \coth^2 al,$$

where H is the heat given per second to one end of the rod, V the excess of temperature of this end over that of the enclosure, h the heat lost per second from 1 sq. cm. of surface when its excess of temperature over that of its surroundings is 1°C., and $a = \sqrt{\frac{hp}{qk}}$.

If l is large the equation reduces to the very simple form $k = \frac{H^2}{pqhV^2}$, this latter equation being employed in nearly every case.

Experimental details and the method of working out the results are given, and it is shown that the thermal conductivity of non-metals can also be determined in the same way.

Electrical conductivities of the same specimens have also been measured, and, for purposes of comparison with electronic theories, the values of k/KT have been worked out, where k =thermal conductivity, K =electrical conductivity and T =absolute temperature.

DISCUSSION.

Prof. C. H. LEES thought the method extremely useful and simple. The accuracy obtained was quite sufficient to allow of verification of the ionic theories on which the author touched. His figures differed sufficiently from the theoretical value to show that the theory was only approximate in its present state.

Dr. HARKER asked about the uniformity of temperature in the air space. The apparatus was horizontal, and if the heat supplied to the wire was very great there were bound to be considerable irregularities of temperature in the chamber.

Mr. BARRATT, in reply to Dr. Harker, said there was sometimes a very slow change of temperature of the air in the enclosure, but as the actual experiments, once conditions had become steady, took only a few minutes, the small change in temperature would not matter. Moreover, this change would affect both the hot end of the specimen and the cold one, so that the value of V , which was the object of the measurement, would not be altered. In addition, the heat supplied to the wire was never great enough to raise the temperature of its hottest part more than 10°C . above that of the enclosure. In fact, the greater part of the wire was at practically the same temperature as that of the water jacket.

XXXVI.—*Some Investigations on the Arc as a Generator of High Frequency Oscillations.* By F. MERCER, B.Eng.

COMMUNICATED BY PROF. WILBERFORCE.

RECEIVED JUNE 3, 1914.

Introduction.

THESE investigations were undertaken in the first instance with the view of determining what effect an increase on the gas pressure in the containing vessel would have on the power and efficiency of a copper-carbon arc when used as a generator of high-frequency oscillations.

A series of experiments on the high-frequency arc were published in the "Bureau of Standards" for May, 1907, by Mr. L. W. Austin. One of the most interesting points in the Paper was that dealing with the effect on the high-frequency current of increasing the pressure of gas surrounding the arc. Consideration of the underlying principles leads one to the conclusion that the effect of this should be to increase the amount of useful power that could be taken from the high-frequency circuit, and Mr. Austin's work shows that this actually does occur. A curve connecting the pressure in atmospheres and the secondary current in amperes shows that up to two atmospheres and beyond five the curve is practically flat, but in the intermediate portion the shunt current increases enormously, rising from 2.5 amperes at two atmospheres to 12 at five atmospheres. For his experiments Mr. Austin used silver-tipped electrodes, and an atmosphere of compressed air. The electrodes were hollowed out and cooled by means of a running stream of water. The nature of these results leads one to the conclusion that further investigation is necessary, and for this reason the author made a series of similar experiments on the ordinary copper-carbon arc. In the course of these some other properties peculiar to the high-frequency arc were brought to light, and these will be referred to in due course.

Apparatus.

The arc was encased in a cast-iron cylinder, capable of withstanding mechanical pressure of 200 lbs. per square inch. The copper electrode and the brass holder for the carbon electrode pass through the ends of the cylinder, being insulated from them by ebonite, and are held in position by glands lined with

ebonite fitting into annular spaces in the ebonite insulation of the ends. Asbestos packing is placed between the gland and the ebonite of the cover, and can be compressed by tightening up the gland, thus preventing leakage and at the same time allowing either electrode to be moved.

In regulating, the carbon only was moved, the copper electrode remaining stationary.

The main leads (Fig. 1) are taken to a double-pole switch, and the primary circuit completed through the inductance L , the ammeter Aa , the resistance R and the arc. The shunt circuit contains an ammeter As , inductance Ls and capacity C . The condensers used were of the Moscicki type, the respective capacities of the four available being 0.00201, 0.00212, 0.00299, 0.00314 microfarad. The inductance Ls consisted of 2.65 mm. wire wound on a square frame of 80 cm. side. The copper

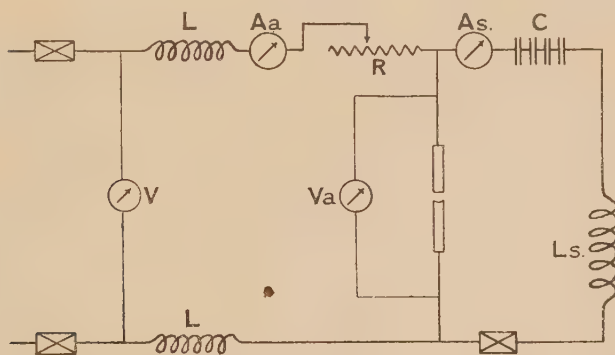


FIG. 1.—DIAGRAM OF CONNECTIONS.

electrode was 19 mm. and the carbon 20 mm. in diameter. A Lorentz wave-meter was used to measure the oscillation frequency. All connections in the shunt circuit were made with thick copper wire in order to reduce its resistance to a minimum.

Experimental Results.

A few preliminary tests were carried out to find the best conditions for producing oscillations. It was found that for steady burning—

(1) The copper electrode should be negative; this conclusion is in agreement with observations made by Barreca.*

* "The Electrician," Vol. LX., pp. 522-523, Jan. 17, 1908.

(2) The magnetic field should be dispensed with, as steadier readings can be obtained without it.

(3) The arc length should be carefully adjusted. If this length is too short the oscillations will not pass, while if too long they are so irregular that no readings can be taken. There is, however, a definite length, varying from about 0.5 mm. to 2 mm., which not only gives steady oscillations but which is least liable to be affected by change in the conditions in the course of any particular experiment.

In the first place experiments were made with a view to determining the effect of arc length on the frequency and magnitude of the current in the shunt circuit. As it is a matter of considerable difficulty to measure accurately the distance between the two electrodes, and knowing that the arc current

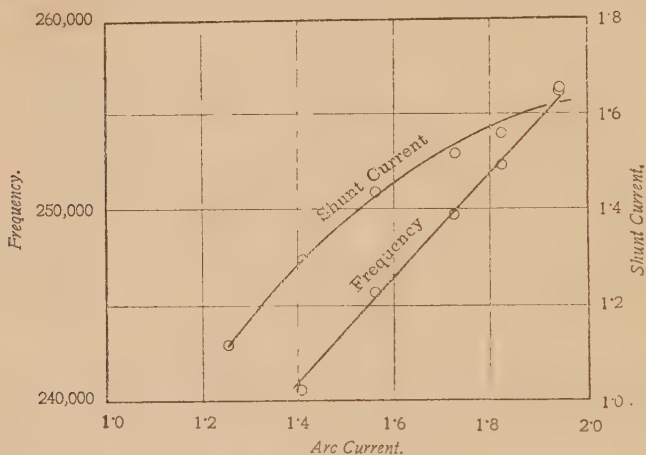


FIG. 2.—CURVES OBTAINED BY ALTERING THE ARC LENGTH AND TAKING READINGS OF ARC CURRENT, SHUNT CURRENT, AND FREQUENCY.

increases as the arc length is diminished and vice versa, we can deduce from the curves in Fig. 2, which are obtained by altering the arc length and taking simultaneous readings of arc current, shunt current and frequency, that both the magnitude and frequency of the shunt current decrease as the arc length is increased. Now, with a copper-carbon arc it is a practical impossibility to ensure that the arc length should remain constant, even for a short time, owing to slight irregularities in the structure of the carbon, and to its burning away, though the latter defect can be remedied to a certain extent by slowly

rotating the carbon. Therefore, even with the most carefully prepared arc of this description, the values of frequency and shunt current are continually altering. By taking certain precautions before any experiment, however, these variations can be cut down to such an extent that their effect is hardly appreciable.

Similar curves can be obtained by altering the resistance in series with the arc. In this case as the current through the arc increases the frequency increases and vice versa.

These facts are borne out by the formula

$$f = \frac{1}{2\pi} \sqrt{\frac{1}{L \cdot C} - \frac{R^2}{4L^2}},$$

where

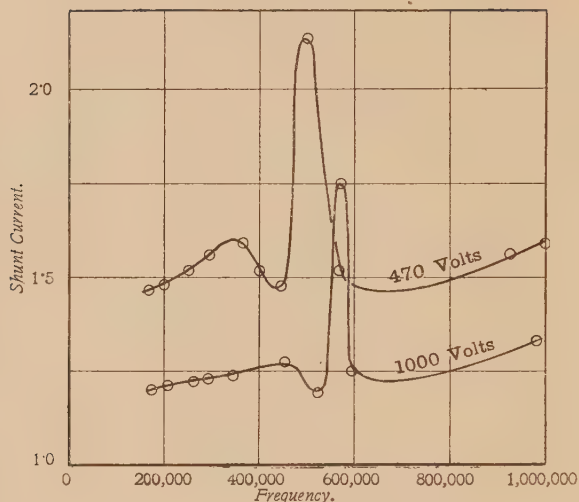
f = frequency of oscillations,
 L = inductance of circuit in henries,
 C = capacity in farads,
 R = resistance in ohms,

since it is fairly safe to assume that increasing the arc length or decreasing the current through the arc would increase its resistance and vice versa.

Effect of Ratio of Self-induction to Capacity.

It has been shown that the ratio of self-induction to capacity has an important influence on the production of oscillations. In order to examine this effect the following experiment was carried out. Keeping the capacity constant the inductance was slowly decreased one turn at a time. The curve connecting frequency with the current in the shunt circuit was found to be of the nature of a resonance curve, examples of which are shown in Figs. 3 and 4.

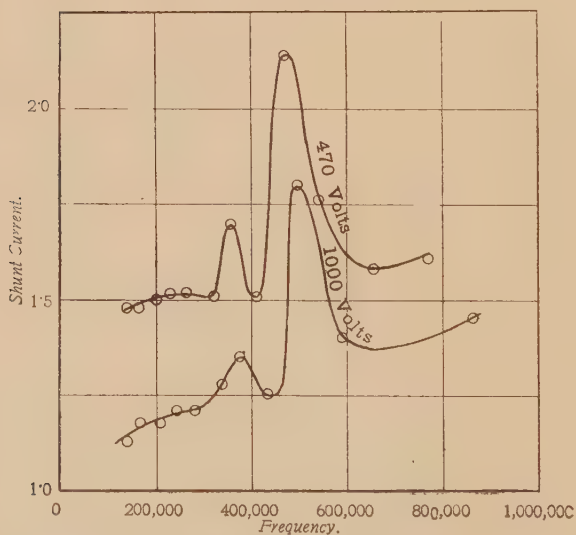
With fairly high capacities in the shunt circuit a gradual increase in current was observable as the inductance was decreased, no apparent maxima occurring. This state of affairs might have been expected, for, in decreasing the inductance, the resistance of the shunt circuit is also decreased by a certain amount. This resistance naturally has considerable influence on the strength of oscillations, as has been shown experimentally by Austin, an increase in resistance giving a decreased current and vice versa. However, in Figs. 3 and 4, in which condensers of smaller capacity were employed, the shunt current had decided maxima, two or even three of these occurring in certain cases. With a given capacity in the shunt circuit



Constant capacity in shunt circuit = 0.0012 mfd.

FIG 3.—CURVES SHOWING THE EFFECT OF ALTERING INDUCTANCE, KEEPING CAPACITY CONSTANT.

Arc current at	470 volts	= 1.4 amperes.
" "	1,000 "	= 1.46 "
" volts "	470 "	= 62 "
" "	1,000 "	= 74 "



Constant capacity in shunt circuit = 0.00201 mfd.

FIG 4.—CURVES SHOWING THE EFFECT OF ALTERING INDUCTANCE, KEEPING CAPACITY CONSTANT.

Arc current at	470 volts	= 1.4 amperes.
" "	1,000 "	= 1.48 "
" volts "	470 "	= 62 "
" "	1,000 "	= 65 "

there is then a definite value of inductance which gives high-frequency oscillations of maximum power. The inductance necessary to give this effect, and the corresponding shunt current, and the frequency, are plotted against capacity in Fig. 5. The steadiness of the oscillations depends on the values of the inductance and capacity, being very poor with either small capacity or small inductance. In order to obtain the conditions for maximum output from the arc, one or other of these two quantities has to be small and consequently steadiness of

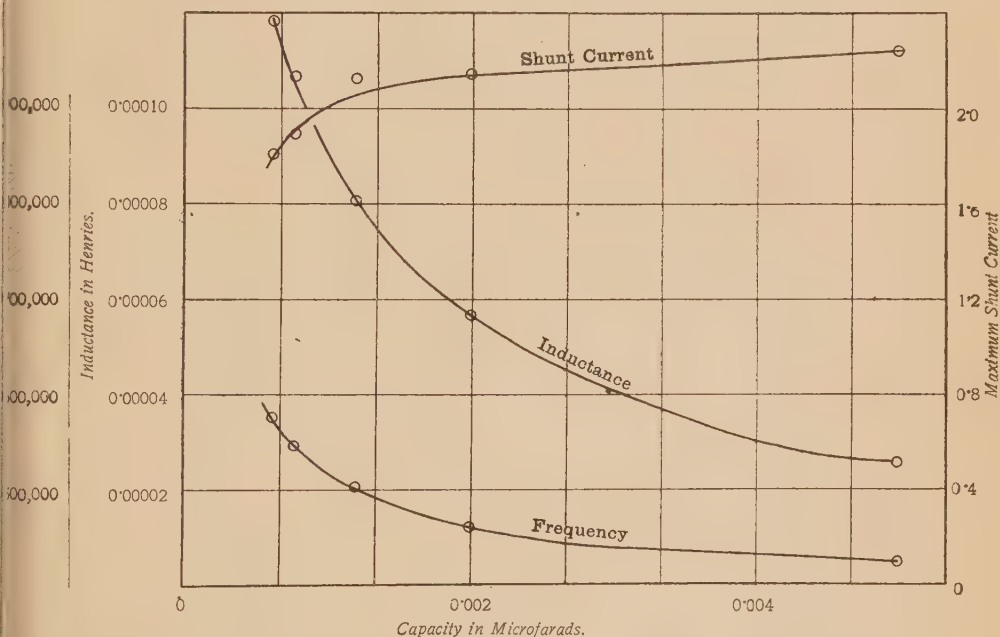


FIG. 5.

CURVES GIVING THE CONDITIONS FOR MAXIMUM SHUNT CURRENT FOR DIFFERENT CAPACITIES.

operation is sacrificed to output. The two curves shown in Figs. 3 and 4 are for 470 and 1,000 volts respectively. It will be seen that the two are similar, except that they reach their maxima at different frequencies. This, however, might be accounted for by differences in the arc length and current.

Effect of Gas Pressure.

The experiments carried out to determine the effect of gas pressure were not conclusive. The maximum electrical

pressure available was about 3,000 volts (direct current), which was obtained by connecting a 2,000 and a 1,000-volt generator in series. The maximum current given by the former was 1 ampere. When using 1,000 volts and over the

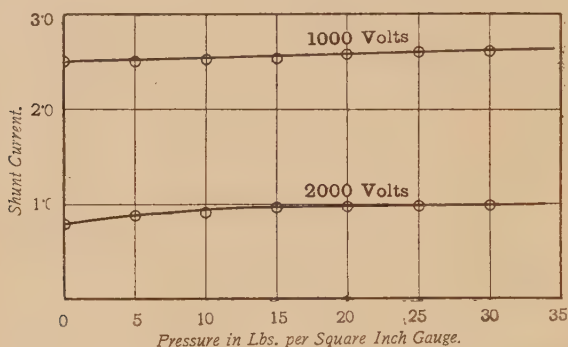


FIG. 6.—CURVE SHOWING THE EFFECT OF ALTERING THE GAS PRESSURE.

copper electrode was earthed. At the lower voltages (230 and 460 volts) it was found that altering the gas pressure had little or no effect. At 1,000 volts, however, a slight rise in current

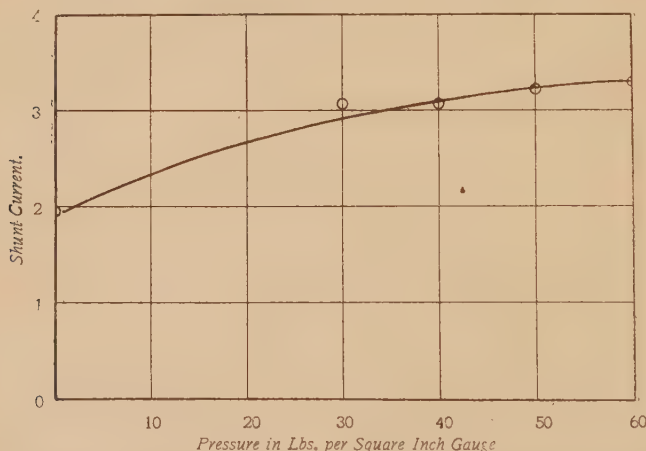


FIG. 7.—CURVE SHOWING THE EFFECT OF ALTERING THE GAS PRESSURE.

could be observed, while at 2,000 volts the curve between shunt current and gas pressure was comparatively steep. When a pressure of 3,000 volts was tried, no steady readings could be obtained. However, a comparison of the results obtained with

230, 460, 1,000 and 2,000 volts leads to the conclusion that the shunt current obtainable increases as the electrical pressure is increased, and it is quite possible that at 5,000 volts, if mean readings could be taken, similar results could be obtained with the copper-carbon arc as were obtained by Austin using silver-tipped electrodes. The magnitude of the effect is dependent on the values of inductance and capacity in the shunt circuit. The plotted results in Fig. 6 were taken with a capacity of 0.0103 mfd. and inductance 0.0002 henry. In Fig. 7 it will be seen that the curve rises more steeply than in either of the previous cases. The inductance in this case was about 0.000025 henry, the capacity being the same, viz., 0.0103 mfd. Otherwise the conditions were similar to those obtaining in the lower curve of Fig. 6.

Conclusions.

(1) There is a definite value of inductance for any given capacity, which gives a maximum current in the shunt circuit.

(2) The effect of increasing the gas pressure becomes more marked as the electrical pressure is increased, but as the gas pressure rises the steadiness of the arc diminishes. The effect is somewhat similar to that obtained by increasing the arc length.

(3) Any effort made to increase the output by the use of a magnetic field, or by altering the arc length or the resistance in series with the arc, is detrimental to the steadiness of operation.

The foregoing experiments were carried out at Liverpool University, and before concluding the author wishes to express his thanks to Prof. Marchant for assistance and advice throughout the work, and also to Mr. N. P. Devonald, B.Eng., who very kindly assisted in carrying out some of the experiments.

ABSTRACT.

This Paper contains the results of a series of experiments on the copper-carbon arc when used as a generator of high-frequency oscillations. The copper electrode was water-cooled, and the whole enclosed in a cast-iron cylinder filled with hydrogen under pressure.

Preliminary experiments were carried out to determine the conditions which would give maximum steadiness of burning, and in the ensuing experiments these conditions were adhered to as rigidly as was possible.

The first experiments deal with the effect of varying the arc length, and also the arc current (by changing the resistance in series with the arc), on the magnitude and frequency of the shunt current. The

deduction is drawn that the effect on frequency arises from a change in the resistance of the arc.

The second experiment refers to the effect on the shunt current of altering the ratio of inductance to capacity. Keeping the latter constant and varying the former a series of curves were obtained, showing that for a given capacity there is a certain value of inductance giving maximum current in the shunt circuit.

Experiments were also made to determine the effect which the pressure of the hydrogen would have on the magnitude of the shunt current. This is dependent on the P.D. used. With small P.D.s the effect is negligible, at 1,000 volts it is just appreciable, and at 2,000 volts a marked increase is apparent. Beyond this it was impossible to obtain readings with any degree of accuracy, as the oscillations were very unsteady.

DISCUSSION.

Prof. FLEMING mentioned that Mr. Bairsto had used an arc under pressure as a source of high-frequency oscillations over a year ago. He then described a more satisfactory method recently used by himself and Mr. Coursey. A number of carbons were fixed vertically to a metal plate which was suspended near the bottom of an iron vessel containing heavy oil. The carbons just project above the oil. Over each carbon tip is suspended a copper cylinder, the walls of which go down into the oil, and the thick top of which is perforated. Arcs are struck by lowering the cylinders until they make contact with the carbons, and then raising them with a screw. In this way very steady oscillations were obtained.

Mr. DUDDELL said that the Society was indebted to Dr. Marchant, who had come from Liverpool purposely to read Mr. Mercer's Paper. Many of the observations mentioned in the Paper were common knowledge to those who had worked with arcs. He would like, however, to call attention to the peculiar shape of the curves connecting the current in the shunt circuit and the frequency which the author had obtained, and in this connection he would like to ask the author whether the oscillations were of what are known as the first or the second type. If the latter were the case, it might be possible to explain the peaks on the curve, and that their position would probably depend on the relation between the self-induction in the main circuit and the capacity in the shunt circuit.

Mr. E. H. RAYNER asked what was the source of supply, and whether large amounts of inductance and resistance were used between the arc and source of power, as it might be that the character of the curves obtained might be modified if the "ballasting" in this part of the circuit were small. He also asked whether any attempt was made to load up the oscillating circuit with resistance to represent power radiated or otherwise employed. One would expect some modification in the results as compared with a "frictionless" oscillating system.

Prof. MARCHANT, in reply, expressed his interest in Prof. Fleming's generator. The author had found that when the power was too great the arc became unstable. He thought Mr. Duddell's explanation was probably correct, as the oscillations were of the second type. It was noticed that with a big capacity, the peak was considerably less pronounced. There was a constant inductance in the circuit, and the current was reduced by adding resistance.

Mr. F. MERCER (communicated) stated in reply to Mr. Rayner that the source of supply was, for the low-voltage experiments, the supply mains, and, for the higher voltage experiments, two high-tension direct-current generators giving pressures up to 3,000 volts when connected in series. The inductance in the primary circuit was about 1 henry and the resistance 290 ohms when working at 470 volts, and 630 ohms when working at 1,000 volts. No additional resistance was included in the secondary circuit.

*Report of the Committee on Nomenclature and Symbols.**

The Committee consists of Prof. Sir J. J. Thomson, O.M., F.R.S. (President), Prof. H. L. Callendar, F.R.S., A. Campbell, Esq., B.A., Dr. C. Chree, F.R.S., Prof. G. Carey Foster, F.R.S., Dr. W. Eccles (Secretary and Convener), Sir George Greenhill, F.R.S., Dr. Alexander Russell, M.A., Prof. the Hon. R. J. Strutt, F.R.S., Prof. S. P. Thompson, F.R.S., Dr. W. Watson, F.R.S.

There have been three meetings—namely, on March 27th, April 24th and June 11th—and the following recommendations have been arrived at respecting electric, magnetic and certain associated quantities.

TYPOGRAPHICAL.

Capitals and Small Letters.—For electrical quantities varying harmonically, capitals should stand for the amplitude and small letters for the value at any instant.

Greek Letters.—Where possible, Greek letters should be used for angles and for specific quantities.

Subscripts.—The use of subscripts for components of vectors should be discouraged. As a general rule subscripts should be avoided.

Abbreviations for Names of Units.—Ordinary type should be used for the symbols of units, and not Clarendon.

NOMENCLATURE.

<i>Terms in common use.</i>	<i>Recommendation.</i>
Magnetic force, magnetic field. .	None.
Intensity of magnetisation . . .	”
Magnetic induction, magnetic flux density	Retain both terms at present.
Inductance, coefficient of self-induction	Urge use of “ inductance.”
Permeability, inductivity . . .	Urge use of “ permeability.”
Electric force, field, intensity ..	None.
Electric polarisation, displacement, electric flux-density	Disuse displacement.
Specific inductive capacity, dielectric constant, dielectric coefficient, permittivity	Urge use of electric inductivity and do not use permittivity.
Capacity, permittance	Do not use permittance.
Dielectric strength, dielectric rigidity and electric strength	Do not use dielectric rigidity.
Specific resistance, resistivity. .	Urge use of resistivity.
Electromotive force, potential difference, voltage	No term preferred.

* Published by the Council with a view to a possible general discussion at a future date.

SYMBOLS.

<i>Quantity.</i>	<i>Recommendation.</i>
Base of Napierian logarithms ..	e rather than ε .
Electric charge	Q, q
„ force	None.
„ inductivity	κ
„ polarisation	P
„ capacity	C, K
Voltage	V, v, E, e
Current	I, i
Magnetic pole	m } For general use.
„ force	H } Symbols used for
„ induction	B } terrestrial magnetism
„ moment	M } and atmospheric
Permeability	μ } electricity given below.
Magnetic flux	F, Φ
Intensity of magnetisation	I
Susceptibility	None.
Self-inductance	L, l
Mutual inductance	M, m
Reactance	X, x
Impedance	Z, z
Resistance	R, r
Resistivity	ρ
Conductance	G
Conductivity	γ
Energy and work	W, w
Power	P
Period	T
Frequency	n, f
Decay coefficient	b
Logarithmic decrement	δ

To illustrate the meanings of these terms the following notes are adopted :—

The period or periodic time of a simple harmonic vibration is the time elapsing between two successive transits in the same direction through the same point. If the equation of the S.H.M. be taken, $v = V \cos \omega t$,

The period is $T = 2\pi / \omega$,

The frequency is $n = \omega / 2\pi$.

The simplest kind of damped vibration may be indicated by the equation $x=e^{-bt} \cos \omega t$.

The period is $T=2\pi/\omega$,

The frequency is $n=\omega/2\pi$,

The decay coefficient is b .

The *logarithmic decrement* is $\delta=bT$, or, in words, is the natural logarithm of the ratio of any value of the function x to its value when t is increased by T .

The *damping factor* is e^δ ; it is the ratio of the values of the function at the times t and $t+T$.

SYMBOLS FOR MULTIPLES AND SUBMULTIPLES.

<i>Multiple or submultiple.</i>		<i>Name.</i>		<i>Symbol.</i>
10^3	Kilo	k
10^{-3}	Milli-	m
10^{-6}	Micro-	μ
10^{-9}	Millimicro-	$m\mu$
10^{-12}	Pico-	p or $\mu\mu$

The usage of $\mu\mu$ as an abbreviation for 10^{-9} metre is undesirable.

SYMBOLS FOR UNITS.

<i>Unit.</i>	<i>Symbol.</i>	<i>Unit.</i>	<i>Symbol.</i>
Ampere	A	Watt-hour	Wh
Volt	V	Volt-ampere	VA
Ohm	Ω	Ampere-hour	Ah
Coulomb	C	Milliampere	mA
Joule	J	Kilowatt	kW
Watt	W	Kilovolt-ampere	kVA
Farad	F	Kilowatt-hour	kWh
Henry	H		

PRINTING OF LARGE NUMBERS.

In printing numbers greater than 100 the method of marking thousands by spaces instead of by commas should be adopted. For example :

1 234 567 instead of 1,234,567.

STATEMENT ON THE DEFINITION OF CAPACITY.

(a) *Capacity of a Conductor* (original definition).

The ratio of the charge to the potential of a conductor when

at a great distance from other conductors is constant, and is called its capacity.

For example, the capacity of a sphere is κr , and of two equal spheres in contact is $2\kappa r \log 2$, where κ is the electric inductivity of the homogeneous medium in which they are immersed and r is the radius of each sphere.

(b) *Capacity of a Conductor* (Maxwell).

The capacity of a conductor is the ratio of its charge to its potential when all neighbouring conductors are earthed. If the position of neighbouring conductors or insulators be altered, then, in general, the value of this capacity will also be altered.

(c) *Capacity between Two Conductors*.

In everyday work the capacity between two insulated conductors is measured by giving a positive charge to the first conductor and an equal negative charge to the second and finding the ratio of the charge on the first conductor to the difference of potential between them.

Provided that the two conductors and all neighbouring conductors are initially uncharged this ratio is constant. It is often of great value in practical work. In a good many cases its value can be computed.

The formal definition is as follows :—

Let two conductors be given in a field comprising dielectrics and other conductors which are uncharged. Then, if the two conductors have charges $+Q$ and $-Q$, and if the difference of their potentials is V , the ratio Q/V is independent of Q , and is called the *capacity of the field round the two conductors*, or simply the *capacity between the two conductors*.

The symbols C and K are indiscriminately used for (a) or (c).

Notes on Terrestrial Magnetism and Atmospheric Electricity.

By CHARLES CHREE, *Sc.D., LL.D., F.R.S.*

THE following notes are not intended to advocate or criticise, but merely to call attention to notation in common use in terrestrial magnetism and atmospheric electricity, two branches of science which lie rather off the beat of the ordinary physicist.

Terrestrial Magnetism.—The notation in most common use when the earth's magnetic force is resolved into three rectangular components is X to the north, Y to the east and Z

vertically downwards. D (or δ) for declination, H for horizontal force, and V for vertical force are also in very common use; while T (also F and R) for total force, I (also θ) for inclination, N north and W west components are also not infrequently met with. The magnetic moment of a magnet is usually referred to some standard temperature and to an imaginary zero field. The former fact is usually represented by the use of a formula such as $m_t = m_0(1 - qt - q't^2)$, where m_0 refers to the standard temperature (usually $0^\circ\text{C}.$), m_t to the existing temperature, q and q' being "temperature coefficients" determined by experiment. When the horizontal force magnet is in the magnetic meridian at a place where H represents the horizontal field, its temporary moment is usually represented by an expression of the form $m + \mu H$. It should be noticed that μ is here used in a somewhat different sense to that usually employed; the volume of the magnet (which is not generally known accurately) is not explicitly introduced. What the user generally knows is the mass of the heterogeneous body composed of magnet, stirrup, glass scale and lens. μ is occasionally used also to denote pole strength—a quantity which there is seldom occasion to introduce explicitly. For the "pole distance," which is in more common use, $2l$ seems the most usual notation. $2l$ (or l) is also sometimes used for pole distance, but more usually for the total length of the magnet. The force exerted by a bar or cylindrical magnet at a point situated in the prolongation of its axis at a distance r from its centre is generally expressed in the form $2mr^{-3}(1 + Pr^{-2} + Qr^{-4})$, where P and Q are termed "distribution constants," of which Q is not infrequently assumed to be zero. The angle made with the magnetic meridian by one horizontal magnet deflected by another is generally called u . The moment of inertia of a magnet and its appurtenances is usually represented by K or I.

Owing to the practice of taking transit observations of the horizontal force magnet at intervals of 5 (sometimes 3 or 7) semi-vibrations, the use of T to denote *half* the period of a complete vibration is almost universal.

In connection with minor changes of magnetic force—*e.g.*, in the ordinary diurnal variation—the unit of force in almost universal use is $1\gamma \equiv 1 \times 10^{-5}$ C.G.S. It is also not infrequently used in connection with the complete value of a magnetic element—*e.g.*, the horizontal force at present in London is approximately $18\,500\gamma$.

When diurnal inequalities are expressed in Fourier series,

the two alternative forms are usually embodied in the following notation :—

$$c_1 \sin (t+a_1)+c_2 \sin (2t+a_2)+\dots$$

$$a_1 \cos t+b_1 \sin t+a_2 \cos 2t+b_2 \sin 2t+\dots,$$

where t —used as an abbreviation for $15^\circ \times$ (time since midnight in hours)—represents the hour angle in the 24-hour term, $2t$ the hour angle in the 12-hour term, and so on.

The dependence of the range of the diurnal inequality on sun-spots is usually expressed by Wolf's formula $R=a+bS$, where R is the magnetic range, S the sun-spot frequency (after Wolf and Wolfer), and a, b constants.

Magnetic Observations at Sea.—A terminology of a very hybrid character peculiar to navigators has considerable vogue. ζ' represents the course of the ship as shown by the ship's own compass, ζ being the course that would be shown by an ideal compass unaffected by ship's iron.

The formula in common use is—

Deviation of compass due to ship's iron

$$=A+B \sin \zeta'+C \cos \zeta'+D \sin 2\zeta'+E \cos 2\zeta'+\dots$$

The ordinary British navigator regards the coefficients A, B , &c., as improperly used unless attached as above. He also regards as natural symbols—

$\lambda \equiv$ (horizontal component on board)/(what horizontal component would be if no iron present).

$\mu \equiv$ (vertical component on board)/(what vertical component would be if no iron present).

θ' and θ are commonly used to represent the inclination as observed on board ship and as corrected for ship's iron respectively.

Special meanings are also attached to various other letters—e.g., P, s, g, c —some of which are rather ambiguous in their dimensions owing to the old practice of taking magnetic force at Greenwich as unity. The use of magnetic “variation” for declination is still very common.

Atmospheric Electricity.— C (or K) for capacity, V for potential and i for current are in common use. The potential gradient is most often measured in volts per metre of height.

The use of suffixes $+$ and $-$, referring respectively to plus and minus ions, is very usual. Thus we have ε_+ and ε_- (ε

being ionic charge), λ_+ and λ_- (λ being conductivity), n_+ and n_- (n being number of ions in the atmosphere per cubic centimetre).

The charge on free ions is most often given in electrostatic units, and then generally refers to a cubic metre of air. The notation varies, I (I_+ and I_-), J (J_+ and J_-), and E (E_+ and E_-) being all met with.

The notation Q for I_+/I_- is not unusual.

n is usually restricted to the light or mobile ions, having mobilities (*i.e.*, velocities corresponding to a potential gradient of 1 volt per centimetre) of the order of 1 cm. per second. The mobilities of these ions are denoted sometimes by u (u_+ and u_-), sometimes by v (v_+ and v_-).

N and V are occasionally used to denote the number and mobility of the Langevin (large) ions.

The notation a (a_+ and a_-), $\bar{a} = \frac{1}{2} (a_+ + a_-)$ and $q = a_-/a_+$ was at one time in very common use for electrical "dissipation" as measured by Elster and Geitel's apparatus.

There was also extensive use of A for the radio-activity of the atmosphere in the arbitrary unit employed by Elster and Geitel (A=1 when 1 metre of wire exposed under high negative potential for two hours to the atmosphere and then introduced inside special type of charged electroscope reduced the potential by 1 volt per hour).

XXXVII. *Production of Very Soft Röntgen Radiation by the Impact of Positive and Slow Cathode Rays.* By Sir J. J. THOMSON, O.M., F.R.S.

[ABSTRACT.]

RÖNTGEN and his pupils had always held that light waves were identical in nature with electrical waves produced by mechanical means, but there was a gap, on which very little work had been done, between the longest infra-red radiation and the shortest electrical wave that could be mechanically produced. He believed the investigation of this gap to be essential to the proper study of the constitution of the atom. The work already done on X-rays had demonstrated the existence of two separate rings of electrons in the atom, one within the other. These rings were responsible for the K and L types of radiation respectively. The L radiation was so much softer than the K that if a third ring of electrons existed, the radiation from which was proportionately softer than that of the L type, this radiation would fall well within the gap already mentioned.

In the first experiment described a special form of discharge tube was employed. The positive rays passed through a tubular perforation in the cathode and impinged obliquely on a metal target. A photographic plate of the Schumann type was situated at the further end of a branch tube in such a position that no solid obstacle interposed between the target and the plate. When the discharge passed between the electrodes the photographic plate was affected. The application of an intense transverse electrostatic field between two metal plates situated between the cathode and the target completely stopped the effect, showing that this was not due to stray radiation *reflected* from the target, since, while charged particles would be swept to one side, radiation would not be affected by the field. Hence the passage of positive particles from the cathode to the target was essential. On the other hand, a strong transverse electrostatic field in the branch tube had no effect, showing that a radiation was passing between the target and the plate, which was not, therefore, merely affected by positive particles rebounding down the side tube after impact on the target.

The properties of this radiation were intermediate between ordinary X-rays and Schumann waves. They were susceptible to reflection by metal surfaces, and their penetrating power was very small. They were completely stopped by the finest collodion film obtainable.

It was shown that the quality of the radiation did not depend on the energy of the moving particles which gave rise to it, but on the velocity. Hence equally soft rays should be produced by cathode particles if these were travelling as slowly as the positive rays. A discharge tube was constructed in which the cathode rays, leaving the cathode with the ordinary velocity, could be subjected to a retarding electrostatic field of variable strength before impinging on the target. In this way the velocity of impact could be varied over a large range, and radiations were obtained varying in quality from ordinary hard X-rays to the so-called Schumann waves. It was hoped by the study of these radiations to be able to determine not only the number of rings of electrons within the atom, but the number of electrons in each ring.

DISCUSSION.

Sir OLIVER LODGE expressed the opinion that the work just described was of far-reaching importance, and he felt confident that the results warranted the anticipation that further work would confirm the explanation foreshadowed by the President. The full meaning of the experiments and the way in which they enabled us to estimate the number of rings in the atom was given to some extent in previous Papers by the President and others. Eventually we would understand the argument more fully, and in this way the unravelling of the secret of the atom would be materially advanced.



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PROCEEDINGS
AT THE
MEETINGS OF THE PHYSICAL SOCIETY
OF LONDON.
SESSION 1913-1914.

October 24th, 1913.

Meeting held at the Imperial College of Science.

Prof. C. H. LEES, F.R.S., Vice-President, in the Chair.

The following Papers were read :—

1. The Ice Calorimeter, with remarks on the Density of Ice. By Mr. EZER GRIFFITHS.
 2. An Electrostatic Oscillograph. By Messrs. H. Ho and S. KOTO.
-

November 14th, 1913.

Meeting held at the Imperial College of Science,

Prof. C. H. LEES, F.R.S., Vice-President, in the Chair.

The following Papers were read :—

1. The Thermal Conductivity of Mercury by the Impressed Velocity Method. By Mr. H. REDMAYNE NETTLETON.

2. Polarisation and Energy Losses in Dielectrics. By Dr. A. W. ASHTON.

3. A Lecture Experiment to illustrate Ionisation by Collision and to show Thermoluminescence. By Mr. F. J. HARLOW.

November 28th, 1913.

Meeting held at the Imperial College of Science.

Prof. C. H. LEES, F.R.S., Vice-President, in the Chair.

The following Papers were read :—

1. The Expansion of Silica. By Prof. H. L. CALLENDAR.

2. The Thermal Expansions of Mercury and Fused Silica. By Mr. F. J. HARLOW.

3. An Experimental Method for the Production of Vibrations on Strings. By Prof. J. A. FLEMING.

December 16th, 1913.

The Ninth Annual Exhibition of Physical Apparatus was held at the Imperial College of Science from 3 p.m. to 6 p.m. in the afternoon and from 7 p.m. to 10 p.m. in the evening.

At both the afternoon and evening meetings a discourse was given by Mr. LEWIS BRENNAN, C.B., on "The 'Iridiscope' and some Experiments on Soap Films," and Prof. J. A. FLEMING gave a short demonstration of his method for producing Vibrations on Loaded and Unloaded Strings.

Experimental Demonstrations were given by Mr. W. E. CURTIS, Mr. F. J. HARLOW, Dr. G. W. C. KAYE and Mr. E. A. OWEN, Prof. J. T. MORRIS and Mr. J. F. FORREST, Mr. CLIFFORD C. PATERSON and Mr. B. P. DUDDING, Mr. F. S. PHILLIPS, and Dr. W. WATSON, F.R.S.

The following firms exhibited apparatus : The Bausch & Lomb Optical Co., The Cambridge Scientific Instrument Co. (Ltd.), A. C. Cossor (Ltd.), Crompton & Co. (Ltd.), J. H. Dallmeyer (Ltd.), Elliott Bros., Evershed & Vignoles (Ltd.), Foster Instrument Co., A. Gallenkamp & Co. (Ltd.), Gambrell Bros. (Ltd.),

F. Harrison Glew, Graham & Latham, John J. Griffin & Sons (Ltd.), Phillip Harris & Co. (Ltd.), Adam Hilger (Ltd.), Isenthal & Co., E. Leitz, the Ludgate Wireless Co., Marconi's Wireless Telegraph Co. (Ltd.), Muirhead & Co. (Ltd.), Nalder Bros. & Thompson, Newton & Co., Robt. W. Paul, the Record Electrical Co. (Ltd.), James Swift & Son, the Synchronome Co. (Ltd.), H. Tinsley & Co., Townson & Mercer, the Weston Electrical Instrument Co., and Carl Zeiss (Ltd.).

January 23rd, 1914.

Meeting held at the Imperial College of Science.

Prof. C. H. LEES, F.R.S., Vice-President, in the Chair.

The following Papers were read :—

1. Some Characteristic Curves and Sensitiveness Tests of Crystal and other Detectors. By Mr. P. R. COURSEY.
 2. A Water Model of the Musical Electric Arc. By Mr. W. DUDDLELL.
 3. Further Experiments with Liquid Drops and Globules. By Mr. C. R. DARLING.
 4. Note on Aberration in a Dispersive Medium and Airy's Experiment. By Mr. J. WALKER.
-

Annual General Meeting.

February 13th, 1914.

Meeting held at the Imperial College of Science.

Prof. C. H. LEES, F.R.S., Vice-President, in the Chair.

The Report of the Council was taken as read.

It has been felt that the Report of the Council should, in future, deal with the same period as that covered by the Report of the Treasurer, namely, from January 1st to December 31st in each year. The present

Report of the Council, therefore, covers only the period from the last Annual General Meeting up to December 31, 1913.

During this period there have been held 10 ordinary meetings and two informal meetings. One ordinary meeting was held at King's College and another at University College. The informal meetings were the Annual Exhibition and a visit to the National Physical Laboratory. The average attendance, excluding informal meetings, was 38.

Owing to the improved financial position the Council feel that the Society's field of activity should be increased, and careful consideration has, therefore, been given, during the past year, to the possibility of introducing new features. It has been thought that expenditure might profitably be made upon the issue, from time to time, of reports upon certain subjects of general interest. The first subject selected for the purpose is Radiation. Mr. J. H. Jeans, F.R.S., has expressed his willingness to write the report upon this subject, and to have it complete during the summer.

The Council have also felt that many Fellows would appreciate an occasional or annual lecture by some eminent physicist, and they have accordingly arranged with Prof. R. W. Wood, of Johns Hopkins University, Baltimore, to give the first of these lectures at an early date. This series of lectures will be known as the Guthrie Lectures, in memory of the late Prof. F. Guthrie, through whose efforts the Society was founded.

During the period under review a Committee has been appointed by the Council to consider questions in regard to Nomenclature and Symbols and allied matters, and consists of Prof. H. L. Callendar, Mr. A. Campbell, Dr. C. Chree, Dr. W. Eccles, Prof. G. Carey Foster, Sir George Greenhill, Dr. A. Russell, Prof. the Hon. R. J. Strutt, Prof. S. P. Thompson and Prof. W. Watson, with Dr. Eccles as secretary and convener. At present the Committee is discussing electric and magnetic quantities; but reports on Mathematical and Mechanical Nomenclature and Symbols, so far as these concern physicists, and on Heat are also projected. The reports will be submitted to the Council and will be published for the use of the Fellows.

The Ninth Annual Exhibition of Apparatus by manufacturers was held on December 16th, in the afternoon and evening, the number of Fellows and visitors present being about 650. The number of firms exhibiting was 30. An experimental discourse on "The 'Iridoscope' and Some Experiments on Soap Films" was given by Mr. Louis Brennan, C.B. Prof. J. A. Fleming repeated his experiments on the Vibrations of Loaded and Unloaded Strings, and for the first time a number of experiments were shown by Fellows and others.

The Society has to thank Prof. A. Schuster for a gift of valuable books.

The number of ordinary Fellows on the roll at December 31, 1913, as distinct from Honorary Fellows, was 442; nine new Fellows have been elected.

The Society has to mourn the loss by death of one member of Council, Prof. P. V. Bevan, and three other Fellows, namely, Sir W. H. Preece, G. B. Finch and Prof. J. G. McGregor.

The Report was adopted by the meeting.

The Report of the Treasurer and the Balance-Sheet were presented by the Treasurer.

The total income of the Society again shows a slight increase over the preceding year, but this is mainly due to the Income Tax recovered and to the increased sale of publications. The subscriptions for the year actually show a slight decrease.

The expenditure for the year has decreased. This is accounted for by the fact that in 1912 the Society issued a special number containing the Papers read at the joint meeting with the Optical Convention.

By comparing the balance brought forward on the 1st January, 1913 (less two cheques)—namely, £140. 12s. 2d.—with the balance carried forward (less four cheques) on the 1st January, 1914—namely, £271. 6s. 3d.—it will be seen that the Society has increased its assets during the year by £130. 14s. 1d. This, I think, shows that the finances of the Society are in a very sound condition.

In spite of the above excess of income over expenditure, the total assets of the Society again show a decrease, mainly due to continued depreciation in the market value of the securities.

The Society has again to thank the Manager of Parr's Bank for kindly valuing the securities at the 30th December, 1913, and for supplying the figures which appear in these accounts.

The liabilities on account of the Life Composition Funds have decreased during the year owing to the deaths of four life fellows and to no new fellows having compounded for their subscriptions. The balance available in the General Fund of the Society is slightly less than last year.

It will be noted that I have reduced the valuations of the publications of the Society since my estimate of last year. The main reason for this is that, on a re-examination of the stock of the Society, it has been found that one of the early parts appears to be missing. If this proves to be the case, it may be necessary to reprint it in order to perfect the complete sets of the "Proceedings of the Society." I have, therefore, deducted a sum in making my estimate which should cover this cost.

The Report of the Treasurer was adopted.

PROPERTY ACCOUNT OF THE PHYSICAL SOCIETY, DECEMBER 31ST, 1913.

ASSETS.		LIABILITIES.	
	£ s. d.		£ s. d.
Subscriptions due, Treasurer's estimate.....	26 5 0	Four Cheques.....	27 7 9
£533 Furness Ry. Co. 3 per cent. Debenture Stock	357 0 0	Life Compositions.....	2,089 10 0
£1,600 Midland Railway 2½ per cent. Preference Stock	956 0 0		
£200 Metropolitan Board of Works 3½ per cent. Consolidated Stock	190 0 0		
£400 Lancaster Corporation 3 per cent. Redeem- able Stock	302 0 0		
£254. 2s. 9d. New South Wales 3½ per cent. Inscribed Stock	237 0 0		
£500 London, Brighton & South Coast Railway Ordinary Stock	515 0 0		
£500 Great Eastern Railway 4 per cent. Debenture Stock	475 0 0		
Balance at Bank	98 14 0		
Ditto on deposit.....	600 0 0		
Publications (Treasurer's Estimate).....	180 0 0	Balance General Fund.....	1,820 1 3
	<u>£3,936 19 0</u>		<u>£3,936 19 0</u>

WILLIAM DUDELL, *Honorary Treasurer.*

Audited and found correct,

HARRY M. ELDER.
THOMAS H. BLAKESLEY.

LIFE COMPOSITION FUND.

	£	s.	d.
174 Fellows paid £10	1,740	0	0
3 Fellows paid £15	45	0	0
4 Fellows paid £21	84	0	0
7 Fellows paid £31. 10s.	220	10	0
	<hr/>		
	£2,089	10	0

NOTE.—Four Fellows who paid £10 deceased during year 1913.

Audited and found correct,

WILLIAM DUDDLELL, *Honorary Treasurer.*

HARRY M. ELDER.
THOMAS H. BLAKESLEY.

The Election of Officers and Council then took place, the new Council being constituted as follows :—

President.—Prof. Sir J. J. THOMSON, O.M., D.Sc., F.R.S.

Vice-Presidents, who have filled the Office of President.—Prof. G. C. FOSTER, D.Sc., LL.D., F.R.S. ; Prof. W. G. ADAMS, M.A., F.R.S. ; Prof. R. B. CLIFTON, M.A., F.R.S. ; Prof. A. W. REINOLD, C.B., M.A., F.R.S. ; Prof. Sir ARTHUR W. RÜCKER, M.A., D.Sc., F.R.S. ; Sir W. DE W. ABNEY, R.E., K.C.B., D.C.L., F.R.S. ; Prin. Sir OLIVER J. LODGE, D.Sc., LL.D., F.R.S. ; Prof. SILVANUS P. THOMPSON, D.Sc., F.R.S. ; R. T. GLAZEBROOK, C.B., D.Sc., F.R.S. ; Prof. J. PERRY, D.Sc., F.R.S. ; C. CHREE, Sc.D., LL.D., F.R.S. ; Prof. H. L. CALLENDAR, M.A., LL.D., F.R.S. ; Prof. A. SCHUSTER, Ph.D., Sc.D., F.R.S.

Vice-Presidents.—Prof. T. MATHER, F.R.S. ; A. RUSSELL, M.A., D.Sc. ; F. E. SMITH, R. S. WHIPPLE.

Secretaries.—W. R. COOPER, M.A. ; S. W. J. SMITH, M.A., D.Sc., F.R.S.

Foreign Secretary.—R. T. GLAZEBROOK, C.B., D.Sc., F.R.S.

Treasurer.—W. DUDDELL, F.R.S.

Librarian.—S. W. J. SMITH, M.A., D.Sc., F.R.S.

Other Members of Council.—W. H. ECCLES, D.Sc. ; Sir R. A. HADFIELD, F.R.S. ; Prof. G. W. O. HOWE, M.Sc. ; Prof. J. W. NICHOLSON, M.A., D.Sc. ; Major W. A. J. O'MEARA, C.M.G. ; C. C. PATERSON ; Prof. O. W. RICHARDSON, M.A., D.Sc., F.R.S. ; Prof. the Hon. R. J. STRUTT, F.R.S. ; W. E. SUMPNER, D.Sc. ; R. S. WILLOWS, M.A., D.Sc.

The following Papers were read :—

1. On the Moving Coil Ballistic Galvanometer. By Mr. R. LL. JONES.

2. On Vacuum-tight Lead-Seals for Leading-in Wires in Vitreous Silica and other Glasses. By Dr. H. J. S. SAND.

February 27th, 1914.

Meeting held at the Imperial College of Science.

Prof. Sir J. J. THOMSON, O.M., F.R.S., President, in the Chair.

Prof. G. CAREY-FOSTER gave a short biography of Prof. Frederick Guthrie, to whom the Physical Society of London owed

its initiation, as an introduction to the first GUTHRIE LECTURE, which was then delivered by Prof. R. W. WOOD, of Johns Hopkins University, Baltimore, who lectured on "THE RADIATION OF GAS MOLECULES EXCITED BY LIGHT."

March 13th, 1914.

Meeting held at the Imperial College of Science.

Dr. A. RUSSELL, M.A., Vice-President, in the Chair.

The following Papers were read :—

1. Time Measurements of Magnetic Disturbances and their Interpretation. By Dr. C. CHREE.
 2. On the Ratio of the Specific Heats of Air, Hydrogen, Carbon Dioxide and Nitrous Oxide. By Mr. H. N. MERCER.
 3. The Asymmetric Distribution of the Secondary Electronic Radiation produced by X-Radiation. By Mr. A. J. PHILPOT.
-

March 27th, 1914.

Meeting held at the Imperial College of Science.

Prof. Sir J. J. THOMSON, O.M., F.R.S., President, in the Chair.

The following Papers were read :—

1. A New Type of Thermogalvanometer. By Mr. F. W. JORDAN.
2. An Instrument for Recording Pressure Variations due to Explosions in Tubes. By Mr. J. D. MORGAN.
3. The Direct Measurement of the Napierian Base. By Mr. R. APPELYARD.

May 8th, 1914.

Meeting held at the Imperial College of Science.

Dr. A. RUSSELL, M.A., Vice-President, in the Chair.

The following Papers were read :—

1. Some Gyrostatic Devices for the Control of Moving Bodies. By Dr. J. G. GRAY.
 2. A Graphic Treatment of Cusped Wave-fronts and of the Rainbow. By Mr. W. R. BOWER.
-

May 22nd, 1914.

Meeting held at the Imperial College of Science.

Dr. A. RUSSELL, M.A., Vice-President, in the Chair.

The following Papers were read :—

1. Volatility of Thorium Active Deposit. By Messrs. T. BARRATT and A. B. WOOD.
 2. The Passage of α -Particles through Photographic Films. By Messrs. H. P. WALMSLEY and W. MAKOWER.
 3. On a Null Method of Testing Vibration Galvanometers. By Mr. S. BUTTERWORTH.
 4. Experiments with an Incandescent Lamp. By Dr. S. W. J. SMITH.
-

June 12th, 1914.

Meeting held at the Imperial College of Science.

Prof. T. MATHER, F.R.S., Vice-President, in the Chair.

The following Papers were read :—

1. Note on the Connection between the Method of Least Squares and the Fourier Method of Calculating the Coefficients of a Trigonometrical Series to represent a given Function or Series of Observations. By Prof. C. H. LEES.
2. A Magnetograph for Measuring Variations in the Horizontal Intensity of the Earth's Magnetic Field. By Mr. F. E. SMITH.

3. The Atomic Weight of Copper by Electrolysis. By Mr. A. G. SHRIMPTON.

4. Note on an Improvement in the Einthoven String Galvanometer. By Mr. W. A. APTHORPE.

June 20th, 1914.

Members of the Society, with a number of friends, visited the works of the Cambridge Scientific Instrument Co., after which they were the guests of the Management and Directors at luncheon in the Hall of St. John's College, Cambridge.

In the afternoon a meeting of the Society was held in the Cavendish Laboratory, Sir J. J. THOMSON, O.M., F.R.S., President, in the Chair.

The following Papers were read :—

1. Production of Very Soft Röntgen Radiation by the Impact of Positive and Slow Cathode Rays. By Sir J. J. THOMSON.

2. On the Homogeneity of Atmospheric Neon. By Mr. F. W. ASTON.

Those present were then entertained to tea by Sir Joseph and Lady Thomson.

June 26th, 1914.

Meeting held at the Imperial College of Science.

Dr. A. RUSSELL, M.A., Vice-President, in the Chair.

The following Papers were read :—

1. On Atmospheric Refraction and its Bearing on the Transmission of Electromagnetic Waves round the Earth's Surface. By Prof. J. A. FLEMING.

2. Atmospheric Electricity Observations made at Kew Observatory. By Mr. GORDON DOBSON.

3. Thermal and Electrical Conductivities of Some of the Rarer Metals and Alloys. By Mr. T. BARRATT.

4. Some Investigations of the Arc as a Source of High-frequency Oscillations. By Mr. F. MERCER.

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